

# A Model of Infrastructure Financing\*

Viral V. Acharya<sup>†</sup> Cecilia Parlato<sup>‡</sup> Suresh Sundaresan<sup>§</sup>

September 2020

## Abstract

Infrastructure projects involve multiple parties: government, private sector firms that build and manage, and outside investors who supply financing. Private sector firms need incentives to implement and maintain the projects well; governments may lack commitment not to extort cash flows (for instance, by limiting user fees) from projects once implemented. This double moral hazard problem limits the willingness of outside investors to fund infrastructure projects. To ameliorate these two moral hazards, we show that the optimal design of infrastructure financing features (I) government guarantees to investors against project failure; (II) bundling of development rights for the private sector and investors; (III) tax subsidies to the private parties out of infrastructure externalities; and, (IV) “general obligation” financing in the form of cross-guarantees between high-quality projects and “revenue only” financing without cross-guarantees for low-quality projects. These features are found to be relevant in the practice of infrastructure financing.

---

\*Viral Acharya is grateful for support from the New York Stock Exchange (NSE) of India and New York University Stern School of Business Joint Initiative on the Study of Indian Capital Markets. The views reflected herein are entirely those of authors and not those of the NSE. We would like to thank Patrick Bolton and Gerard Hertig for their comments and suggestions as well as seminar participants at the 2014 Conference of the NSE and NYU Stern Joint Initiative. Abhishek Bhardwaj and Yueteng Zhu provided excellent research assistance.

<sup>†</sup>New York University Stern School of Business. e-mail: vacharya@stern.nyu.edu

<sup>‡</sup>New York University Stern School of Business and NBER. e-mail: cparlato@stern.nyu.edu

<sup>§</sup>Columbia University Graduate School of Business. e-mail: ms122@gsb.columbia.edu

# 1 Introduction

Infrastructure projects involve multiple parties: government, private sector firms that build and manage, and outside investors who supply financing. Infrastructure projects typically also involve high levels of capital investments. These investments are usually front-loaded; the attainment of satisfactory steady-state revenues to repay financiers of such projects often takes multiple periods, and crucially depends on the efforts expended by two of the key players: the government and the private firms that build and maintain the infrastructure. Both parties may have incentives to extort or expend sub-optimal effort for respective private benefits. The private sector firms must be incentivized to implement and maintain the projects well, a standard moral hazard problem in corporate finance. In addition, governments must also credibly commit not to extort cash flows from projects once implemented; the government can opportunistically, for instance, limit user fees such as tariffs or give “toll holidays” to appease the voting public.<sup>1</sup>

This *double moral hazard* problem limits the willingness of outside investors to fund infrastructure projects as they seek to ensure that they earn a fair rate of return on investment. This can manifest both at the *extensive* margin, i.e., some projects may not get funded in the first place, and at the *intensive* margin, i.e., some projects may get funded but only up to a sub-optimal scale. Note that such lack of willingness on the part of investors to fund infrastructure arises even absent any risk-premium considerations; put differently, it arises purely due to the impact of agency problems on the expected cash flows from infrastructure projects. Conversely, the optimal financing of infrastructure projects should address the double moral hazard problem in order to improve the financing of such projects.<sup>2</sup> In this paper we develop a stylized model to explore how such double moral hazard affects the optimal financing of infrastructure projects.

In our model, the private sector can shape the quality of the project with its inputs and will provide them at the efficient level only if it has incentives to do so. Once the payoff of the project is realized, the government cannot commit not to “extort” the project’s cash flows unless it has incentives not to engage in such extortion. Extortion may take the form of coercive diversion of project cash flows, retroactive taxes, restrictions on price-setting, etc., which have direct or indirect benefits to the government at the expense of cash flows left behind for the private sector operator and investors. Hence, there is potential in our model for double moral hazard. Crucially, the provision of inputs by the private sector and the government’s extortion interact in determining the project’s payoffs, and, in turn, affect each sectors’ incentives.

---

<sup>1</sup>With some exceptions, user fees on infrastructure are invariably subsidized at levels well below marginal cost. Alm (2010) notes that the problems that lead to this outcome include inadequate billing and collection procedures, insufficient attention to operations and maintenance, and political constraints. Government extortions are also documented in “Corruption, Political Connections, and Municipal Finance” by Butler, Fauver and Mortal (2009)

<sup>2</sup>For instance, Lewis and Bajari (2014) show through modeling and empirical evidence that moral hazard is a central issue in highway procurement projects. They argue for careful contract design with emphasis on incentives. It is also possible that corruption in government agencies responsible for the oversight and implementation of infrastructure projects can lead to siphoning off resources and using of sub-standard inputs. Liu and Mikesell (2014) provide quantitative estimates of such extortions by public officials in state funded projects in the United States.

The project is funded by private investors who rationally anticipate that the returns they will receive are affected in expectation by the strength of the government-sector and private-sector incentives. To mitigate the losses borne by investors when the project fails and increase funding, the government can offer guarantees in the event that the project fails. The private investors also take into account any provision of such government guarantees while financing the infrastructure project. Such guarantees will expose the government to the risk of project failure and potentially ameliorate the government moral hazard. However, the size of the guarantees is limited by the fiscal constraint that the government faces in making available its balance-sheet for infrastructure finance.

We show that to ameliorate the resulting inefficiencies in feasibility and scale of infrastructure projects, the optimal design of infrastructure financing features the following salient characteristics:

1. **Government guarantees** to investors against project failure. The government can choose to provide such guarantees in the infrastructure contracts, which we assume to be enforceable due to sufficiently high costs of defaulting on its promises. Such guarantees can provide optimal incentives to the government not to extort cash flows, weaken private sector incentives to shirk, and decrease the risk of project failure. This way, guarantees free up project cash flows to provide the private sector incentives to exert effort and to repay investors. The extent of fiscal commitment from its overall balance-sheet that the government can set aside for the provision of such guarantees affects the scale up to which infrastructure projects can be financed by investors.
2. **Bundling of development rights** for the private sector and investors. Infrastructure projects can also lead to private sector opportunities (for example, housing development around a highway, subway or bridge). If the rights to claim such opportunities can be bundled with the infrastructure project, then the private sector's incentive constraint and/or individual rationality constraint of investors are/is relaxed and adverse spillovers from the government moral hazard are reduced.
3. **Tax subsidies** to the private sector and investors funded out of infrastructure externalities. The government can meet the tax subsidies, for instance, by collecting taxes from beneficiaries of the externalities. If such tax subsidies can be credibly provided by the government (unlike its inability to commit to avoiding cash flow extortion), then they help ameliorate the private sector's moral hazard and they improve the ability to repay investors.
4. **Cross-guarantees** between high-quality projects but direct government guarantees (out of its overall balance-sheet) only for low-quality projects. Positive cross-guarantees always improve the incentive of financiers to participate in the project. However, such guarantees decrease the government's willingness to participate in the projects, and when projects have low probability of success, weaken the government's commitment not to extort cash flows. When the return of the projects is high enough,

the government's expected return from the project is high enough to incentivize it to participate and not extort, and cross-guarantees are optimal. The cross-guarantees can be naturally interpreted as "general obligation" (GO) financing as is common in municipal bond issuance in the United States, whereas their absence can be interpreted as "revenue only" (RO) financing where municipal bonds are paid off only from project-specific revenue collections.

Interestingly, tax subsidies, development rights and cross-guarantees, whenever feasible, can potentially make infrastructure projects self-financing in the sense of not requiring direct guarantees from the overall balance-sheet of the government. As we discuss in Section 2, many of the financing arrangements that we derive as optimal in our model are prevalent in infrastructure projects and their financing in practice.

Furthermore, we show in a final extension of the model that when infrastructure projects also involve an early-stage requirement of government clearances (land acquisition, for example, is often the bottleneck in emerging markets), then the term-structure of guarantees in the optimal financing of projects reflects the relative severity of the early-stage government moral hazard relative to the late-stage one relating to cash flow extortion. Specifically, if the early-stage moral hazard is more severe, then the optimal financing of the infrastructure project features a higher government guarantee against the risk of project failure initially, with the guarantee tapering off to a lower level once the project is off the ground.

The double moral hazard we study in this paper is not just present in physical infrastructure projects but it is also prevalent in projects related to the provision of public goods and public health. A clear example is the investment in the development and production of vaccines. Typically, patents or monopoly power are thought to be ex-ante desirable to incentivize the right amount of private effort and investment. However, if for public health reasons the user fees will ex-post be forced to be low by the government, then its anticipation can discourage private investment. To restore the willingness of the private sector to invest, the government can then either commit to paying the difference between the market user fee and the public cap on the fee (which may be hard to implement) or subsidize the investment upfront recognizing its ex-post moral hazard. One way to subsidize such investment is for the government to provide guarantees in the event of the project's failure.

## **Related Literature**

This paper fits as an application of the literature of contracting under agency problems (see Tirole (1992) and Bolton and Dewatripoint (2004) for textbook analyses). More specifically, contracts with double moral hazard are considered, for example, in Cooper and Ross (1985), Eswaran and Kotwal (1985), Romano (1995), among others. These papers analyze double moral hazard problems in the context of product warranties, agricultural risk-sharing arrangements, and wholesale-retailer agreements and focus mostly on the risk-sharing features of the contracts. Surprisingly, in spite of the importance of infrastructure to the long-run

growth of economies there is only a sparse literature focused on providing the agency theoretic foundations of infrastructure financing, let alone the double moral hazard we consider in this paper.

The existing theoretical literature mostly focuses on partnerships between the private and public sectors either as co-owners/co-managers of the projects or as co-investors. For example, Martimort and Sand-Zantman (2006) and Perotti (1995) focus on whether projects should be organized/owned by the government or by the private sector. Martimort and Sand-Zantman (2006) consider the classic infrastructure problem in which the government can deliver a public good or service under public ownership or outsource the activity to the private sector. They examine the optimal delegated management contract when the government has private information about the project's quality and the private sector's effort is not verifiable. These two frictions lead to the following trade-off: project retention by the government signals high project quality at the expense of a lower return for the private sector, which exacerbates its moral hazard. In their model, project retention by the government increases with the quality of the infrastructure project and full privatization emerges only for the worst quality projects.

Relatedly, in Perotti (1995) partial privatization allows the government to credibly signal that it will not behave opportunistically upon privatization (such as decreasing or even eliminating tolls, once the toll-highways are privatized). In the model, the government's type determines whether it will behave opportunistically and, thus, it cannot be incentivized not to misbehave—there is no moral hazard. In this context, privatization serves as a way of revealing the government's private information, its type. In both of these papers, the organizational structure of the project partially resolves a problem of asymmetric information between the government and the private sector. In our model, we abstract from asymmetric information and instead consider moral hazard for the government, which allows us to focus on the double moral hazard that is characteristic of infrastructure projects, as we describe below. To emphasize the implications of the double moral hazard on the financing of infrastructure projects, we take the need for delegated management as given.

Another part of the literature has viewed infrastructure projects as jointly owned investment options. Banerjee, Gucbilmez, Pawlina (2014), for example, provide a real-options framework to investigate the optimal investment timing in the presence of such joint ownership, bargaining and side payments. Medda (2007), argues that in the case of large-scale public-private partnerships, if the guarantees provided exceed the potential financial losses of private sector, it can lead to strategic behavior and lead to problems of moral hazard. These papers focus on the risk-sharing implications of public-private partnerships.

Finally, another strand of literature that is related to infrastructure is the research pertaining to private equity and venture capital. Recent empirical research by Andonov, Kräussl and Rauh (2020) suggests that infrastructure assets display cash flow properties akin to private equity investments. They report that public institutional investors implicitly subsidize infrastructure investments in a significant way. The literature

on venture capital (VC) contract design views the VC firms and the firms seeking capital as forming joint ventures, in which VC firms add human capital, bring in outside investors and management talent, and establish strict management control (Hellman and Puri, 2000). Repullo and Suarez (1998) and Casamatta (2003) examine the contracting arrangements between VC firms and the firms needing capital under double moral hazard. They show that the optimal financing arrangement can resemble the convertible preferred shares often used in practice.

The rest of the paper is organized as follows. In Section 2 we discuss several features of infrastructure projects and their financing in practice. In Section 3 we present our benchmark model with double moral hazard. In Section 4 we extend the benchmark model to consider development rights, general obligation and revenue only financing, respectively. Finally, Section 6 introduces an early stage moral hazard problem for the government and Section 7 concludes. All omitted proofs are in the Appendix.

## 2 Infrastructure Financing in Practice

Over a period of time, governments and private sector firms in many infrastructure investments have come together with varying contractual arrangements to design and execute projects with varying characteristics in terms of externalities for the government and the private parties. To provide an institutional backdrop for our formal analysis, we briefly review some of the contractual arrangements that have been used in different countries.

**Government guarantees:** In the United States the Transportation Infrastructure Finance and Innovation Act (TIFIA) of 1998<sup>3</sup> established a Federal credit program for transportation projects of national or regional significance. The idea behind the Act is to attract private capital and thereby leverage the capital provided by the government at a cost that cannot be matched by the private sector acting alone. TIFIA provides three types of financing arrangements: *secured direct loans* to the sponsors of the project, *loan guarantees* to institutional investors who make loans to the project, and *long-term standby lines of credit* that may be drawn by the sponsors of the project. TIFIA facilities have a relatively low cost, usually tied to the 10-year Treasury rates. Since 1998, TIFIA has provided over \$8 billion credit for highway, transit, and other projects, mainly backed by user fees and tolls.

In France, a two-pronged approach is used to finance infrastructure projects through public sector-private sector partnership (PPP) programs. The French government provides guarantees to bank loans that are directed towards infrastructure projects. This allows commercial banks to provide funding to private sector sponsors of infrastructure projects. Second, the government has established another guarantees program to

---

<sup>3</sup>TIFIA was passed by Congress in 1998 with the goal of leveraging federal dollars and attract private and non-federal capital into transportation infrastructure. See “Transportation Infrastructure Finance and Innovation Act” (2011) for details.

promote debt financing. These guarantees perform two functions. First, they promote the liquidity of the market for bank loans and bonds. Second, through government guarantees, infrastructure projects can be funded at relatively low costs.<sup>4</sup>

Similar contractual arrangements are used in Australia in their PPP programs to fund infrastructure projects. In particular, Australia has a guarantee program to address the funding gap in infrastructure financing. In addition, outright cash subsidies are also provided for some infrastructure projects.<sup>5</sup>

**Externalities:** Infrastructure investments may often require significant acquisition of land and other properties, which may be in private hands. The government may be able to acquire such resources through compensations in the interest of “public good” or positive externalities that may be created through the provision of infrastructural services. In the absence of government initiatives, it is hard to imagine why the private sector will act on its own to make such investments, leading to market failures. An important example in this context is the public health infrastructure. Sufficient capacities of hospital beds, medical equipments, and human capital in healthcare professionals (nurses, doctors, and other specialists) would likely provide the necessary public goods in the event of a pandemic, and yet it is difficult to design incentives ex ante to elicit such investments from the private sector. Further, government and elected officials may derive benefits from the production of such public goods, but private parties (suppliers of capital and firms that build the infrastructure) may not directly benefit from such investments.

**Development rights:** Some infrastructure projects can result in significant development rights of lands and buildings adjoining the project. Future cash flows from such development rights can increase the overall attractiveness to different contracting parties. Gupta et al. (2020) show that the new transit infrastructure project in New York city resulted in significant spillover benefits to local real estate prices through reduction in transit times for commuters in those localities. Some infrastructure projects can also deliver significant development rights that can affect ex ante the way in which the project may be financed and reduce the operating costs of using the infrastructure. Hong Kong Mass Transit Railway Corporation (MRTC) which covers slightly over 200 kilometers with 84 stations and 68 light train stops resulted in significant increase in land and property values close to the stations. Such rights were deemed valuable to all parties in the contracting arrangements. According to one study, during the period 1998-2013, property-related development operations generated nearly twice the amount of money spent on railway line construction.<sup>6</sup>

**Tax subsidies:** A unique innovation is the tax treatment of municipal bonds: the interest income from municipal bonds is tax-exempt from the perspective of private investors, and the bonds were (up until the

---

<sup>4</sup>See “Public and private financing of infrastructure Policy challenges in mobilizing finance”, EIB Papers Volume 15 No 2, 2010.

<sup>5</sup>See “Infrastructure Partnerships Australia: Financing Infrastructure in the Global Financial Crisis,” March 2009.

<sup>6</sup>Land Value Capture Mechanism: The Case of the Hong Kong Mass Transit Railway by Mathieu Verougstraete and Han Zeng (July 2014).

global financial crisis)<sup>7</sup> typically insured by monoline insurance companies. Together, these two features (and the enforcement of contractual obligations, ex-post) have allowed the development of a fairly big municipal bond market, which, as we discuss below, offers a major source of funding of infrastructure projects in the United States.<sup>8</sup>

**General obligation versus revenue-only financing:** Many projects at state and city levels, such as public hospitals, highways, bridges, etc., are funded in the United States through the issuance of municipal bonds. The municipal bond markets is large and diverse with \$4.0 trillion of municipal debt outstanding and over 50,000 issuers. The municipal bonds typically fall into two categories: the so-called general obligation (GO) bonds, which depend on the overall tax revenues of the State or City for cashflow integrity, effectively providing cross-guarantees across different municipal projects; and, the so-called revenue only (RO) bonds, which depend only on user-fees such as tolls. The revenue bonds account for nearly two-thirds of the municipal bond market as measured by their share of the Bloomberg Barclays Municipal Index, as of June 30, 2019.<sup>9</sup> It is an open question, to the best of our knowledge, as to what determines the choice across infrastructure projects between financing with GO bonds (cross-guarantees) and RO bonds (no cross-guarantees).

The following conclusions may thus be drawn about the financing arrangements that one observes in practice for infrastructure projects. First, the government is directly involved either as a guarantor (and occasionally also as a co-lender or as a supplier) of risk capital. This role is played explicitly by the government or its agencies or supranational institutions with implicit or explicit backing. Second, some infrastructure projects (such as low income housing, slum clearance, sanitation, health services, etc.) can deliver “public goods” that may be valued by the government but not necessarily by the private parties. Third, some infrastructure projects can deliver development rights for private parties. These rights can influence the way in which such projects are financed. Finally, cross-guarantees and tax subsidies feature in several municipal bond financing instruments for infrastructure projects.

### 3 Benchmark Model

We first develop a benchmark model with double moral hazard to provide the rationale for why government guarantees would be essential for efficient financing of infrastructure finance. Then, we extend the model to

---

<sup>7</sup>Since the global financial crisis and in the aftermath of monoline insurer default/distress, while monoline insurance guarantees are no longer the norm for municipal bonds, the bonds are increasingly held by the following groups in their order of ownership: Households and Nonprofit Organizations, Money Market Mutual Funds, Mutual Funds, Closed-End Funds, U.S. Chartered Depository Institutions and Banks Brokers and Dealers and Exchange-Traded Funds. See MSRB, “Trends in Municipal Bond Ownership,” (2019).

<sup>8</sup>Given households are the largest holders of municipal debt, the tax treatment of municipal bonds has attracted some attention from researchers such as Green (1993), Ang, Bhansali and Xing (2010), and Longstaff (2011).

<sup>9</sup>See “Why Municipal Revenue Over General Obligation Bonds,” August (2019), by Marques and Barton, BNY Mellon, Investment Management.



highlight the role of other aspects of infrastructure financing.

### 3.1 Setup

We consider an infrastructure project run by a private project operator, to whom we will refer simply as the “private sector” or the “the private sector operator”. The project is constant returns to scale. We denote the scale of the project as  $I$ . The project is risky and has a payoff  $RI > I$  if it is successful and zero otherwise. The private sector can affect the probability of the project’s success. If the private sector exerts high effort and provides a high input, the project’s probability of success is  $p_h \in (0, 1)$ , else it is  $p_l$ , where  $0 < p_l < p_h$ . We denote by  $\Delta p$  the difference in these probabilities:  $\Delta p \equiv (p_h - p_l)$  and we assume that  $p_l R < 1 < p_h R$  so that the project is worth pursuing only if the private sector exerts high effort and provides a high input. Moreover, if the private sector does not exert high effort, it derives a non-pecuniary private benefit of  $BI$ , where  $B > 0$ .

The government can extort the cash flows from the project from the investors and the private sector and keep the project’s return to itself. While the possibility of such extortive behavior remains ex post, the government can commit ex ante to share the returns of the project provided it has the incentives to do so ex ante. To ameliorate its moral hazard and compensate the private investors for their anticipated loss of return in case the government engages in extortive behavior, the government can commit to provide guarantees to the private investors in the eventual project failure (whose likelihood is greater in case of low effort by the private sector operator). While the government cannot be forced not to extort nor to provide guarantees, we assume that once the government commits to a contract, the contract is enforceable. In other words, we think of the default cost for the government being large enough so that the contract is always fulfilled. Moreover, when deciding whether to commit to a contract, the government takes the contract as given.

We denote the payoff of the government if the project succeeds, inclusive of the extorted cash flows and net of the compensation to the private sector if the project succeeds, as  $R_g$ . We assume that the government can provide a guarantee to the private investors of  $K_g I$  in case the project fails. The size of this guarantee is constrained by the government’s fiscal capacity which can never exceed an upper limit  $\bar{K}$ , determined outside of the model by the government’s balance sheet constraints. By providing this guarantee, the government internalizes the downside of the project and has incentives to induce the private sector to supply inputs efficiently. We assume that the fiscal resources  $\bar{K}$  are available to the government after the investment stage and, therefore, cannot be used by the government to invest directly in the infrastructure project. We discuss the case in which the government can use these resources to invest directly at the end of this section.

Therefore, the private investors receive a guarantee  $K_g I$  if the project fails and a payoff denoted as  $R_b I$  if the project succeeds. The residual payoff,  $(R - R_b - R_g)I$ , accrues to the private sector operator and will serve to incentivize it to exert effort. Finally, we assume that both the private investors and the

Private Investors Fund Project	Project Payoff	Private Investor payoff	Private Sector payoff	Government Payoff
p	$RI, R > 0$	$R_b$	$R - R_b - R_g$	$R_g$
1-p	0	$K_g$	0	$-K_g$

Figure 1: State space of possible outcomes and corresponding payoffs for various economic agents under the benchmark model

government require a net rate of return on their respective investments in the project (in case of government, the contingent investment in the form of the guarantee). We normalize the government's gross rate of return to one and denote the private sector's gross rate of return by  $r \geq 1$ .

The state space of outcomes for the projects, project payoffs, and the payoffs to the various parties (the private sector project operator, the private investors, and the government) are summarized in Figure 1.

### 3.2 Analysis

To analyze the properties of the model, we consider in turn the two incentive compatibility constraints (one each for the private sector and the government), the two individual rationality constraints (one each for the private investors and the government), and the government's fiscal constraint.

1. The private sector's incentive compatibility constraint states that the expected payoff from exerting the high effort must not be dominated by the expected payoff (inclusive of the private benefits) from exerting the low effort:

$$p_h [R - (R_b + R_g)]I \geq p_l [R - (R_b + R_g)]I + BI \quad (\text{ICP})$$

or

$$[R - (R_b + R_g)] \geq \frac{B}{\Delta p}.$$

2. If the government extorts, it can still ensure the private sector implements the high probability  $p_h$  provided that  $R_g \leq \bar{R}_g \equiv \left(R - \frac{B}{\Delta p}\right)$ . This upper bound on  $R_g$  leaves sufficient cash flow for the private sector operator to be incentivized to exert effort. Hence, ex post, the government will always extort at least up to this upper bound. In this case there is no residual cash flow left to pay off the investors, i.e.,  $R_b = 0$ . Note also that if the government extorts beyond this upper bound, then the private sector

operator will not exert effort and implement  $p_l$ . Then, the government might as well extort the entire cash flow up to  $R$ . Then, to implement  $p_h$  by the private sector operator, the financing contract needs to satisfy the following incentive compatibility constraint for the government to ensure it will not extort beyond the upper bound  $\bar{R}_g$ :

$$p_h \bar{R}_g - (1 - p_h) K_g \geq p_l R - (1 - p_l) K_g \quad (\text{ICG})$$

or

$$K_g \geq \frac{p_l R - p_h \bar{R}_g}{\Delta p},$$

which after substituting for  $\bar{R}_g$  can be further simplified to

$$K_g \geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right).$$

Note that a higher government guarantee makes it easier to satisfy the incentive compatibility constraint of the government. Intuitively, the higher the guarantee, the more the government cares about the project failing and, hence, the higher its incentive to not extort the whole payoff and thereby make sure the private sector exerts high effort.

3. The private investors, however, must also be left with an adequate share of the project payoff, so that this expected share plus the expected value of the government guarantee compensate the investors for an adequate rate of return on their investment in the project. This yields the private investors' individual rationality constraint:

$$rI \leq p_h R_b I + (1 - p_h) K_g I \quad (\text{IRP})$$

or, using that  $R_b = 0$  when the government extorts,

$$\frac{r}{1 - p_h} \leq K_g.$$

In other words, the government guarantees in the event the project fails have to be high enough to compensate the private sector from not receiving any cash flows from the project when the project succeeds.

4. At the time of investment, the government has to have incentives to provide the guarantees. The individual rationality constraint for the government is that the expected payoff from the project has to be greater than its outside option, which we normalized to zero. Formally,

$$[p_h \bar{R}_g - (1 - p_h) K_g] I \geq 0, \quad (\text{IRG})$$

or

$$(1 - p_h)K_g \leq p_h \bar{R}_g,$$

which substituting for  $\bar{R}_g$  can be further simplified to

$$K_g \leq \frac{p_h \left( R - \frac{B}{\Delta p} \right)}{1 - p_h}.$$

5. Finally, the guarantee that the government provides,  $K_g$ , cannot exceed its fiscal constraint when the guarantee has to be honored:

$$K_g I \leq \bar{K}. \quad (\text{Fiscal-constraint})$$

Note that the fiscal constraint limits the scale of the investment for a given size of the (per-unit) guarantee provided by the government to the private investors. This is natural as absent the fiscal constraint, the government can always ameliorate its moral hazard problem by setting the guarantee to be sufficiently high and any project scale can then be supported. But this is unrealistic in the scenario where the guarantees must be honored in case of project failure.

The objective of the financing problem is to maximize the net present value of the infrastructure project,  $(p_h R - 1)I$ , that is, its expected payoff net of investment (as all other payoffs are simply transfers between the government and the private sector), subject to the constraints above.

Notice that the fiscal constraint always binds and therefore the scale of the project will be given by

$$I = \frac{\bar{K}}{K_g}.$$

Therefore, the optimal contracting problem solves

$$\max_{K_g \geq 0} (p_h R - 1) \frac{\bar{K}}{K_g}$$

subject to

$$\begin{aligned} K_g &\geq \frac{r}{1 - p_h}, \\ K_g &\leq \frac{p_h \left( R - \frac{B}{\Delta p} \right)}{1 - p_h}, \text{ and} \\ K_g &\geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right), \end{aligned}$$

where we used that the government's extortion problem leads to  $R_b = 0$ ,  $R_g = \left(R - \frac{B}{\Delta p}\right)$ , and that the residual payoff left is enough to incentivize the private sector to exert effort. The proposition below shows the inefficiencies resulting from the double moral hazard problem.

**Proposition 1. (Inefficiency of double moral hazard)** *The double moral hazard affects both the feasibility and the scale of the infrastructure project:*

a. **(Feasibility)** *There exist thresholds  $\underline{\Gamma}$  and  $\bar{\Gamma}$  such that:*

i. *If  $\left(R - \frac{B}{\Delta p}\right) < \underline{\Gamma}$ , the project is not funded even in the absence of any government moral hazard.*

ii. *If  $\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) < \bar{\Gamma}$ , the project is not funded in the presence of government moral hazard.*

iii. *Otherwise, the project is funded only if a government guarantee is provided to investors.*

b. **(Scale)** *If the project is funded, the optimal guarantee (scale) is weakly increasing (decreasing) in the severity of the moral hazard measured by  $\frac{B}{\Delta p}$  and decreasing (increasing) in the return of the project  $R$ . The scale of the project is limited, i.e.,  $I = \frac{\bar{K}}{K_g}$ .*

The proposition above shows that the double moral hazard problem imposes inefficiencies in the financing of infrastructure projects, either by rendering the projects infeasible (extensive margin) or by limiting their scale (intensive margin). The type and magnitude of these inefficiencies depends on the size of the project's return  $R$  relative to the severity of the double moral hazard, which is measured by  $\frac{B}{\Delta p}$ . The ratio  $\frac{B}{\Delta p}$  measures the opportunity cost for the private sector of exerting effort relative to the increase in the project's probability of success if effort is high. When  $\frac{B}{\Delta p}$  is high, it is tempting for the private sector to reap the private benefits of providing low effort. In this case, the private sector requires a high payoff to be incentivized to exert high effort. In turn, this implies that the payoff to the government from not extorting,  $\left(R - \frac{B}{\Delta p}\right)$ , is low and, hence, high guarantees are needed to incentivize the government not to extort, making the scale of the project small. Therefore, a high ratio  $\frac{B}{\Delta p}$  implies that *both* moral hazard problems in the economy are severe.

When the return of the project is low relative to the severity of the moral hazard, i.e., when  $\underline{\Gamma} \equiv \frac{r}{p_h} > \left(R - \frac{B}{\Delta p}\right)$ , it is not possible to incentivize the investors to fund the project even if the government could commit to giving its entire payoff from the project to the investors. In this case, the project is not funded, even in the absence of government moral hazard.

When the return of the project is high enough to be funded in the absence of government moral hazard, i.e.,  $\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right)$ , the government's extortion imposes further limits on the project's feasibility. In particular, for returns of the project such that  $\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) < \bar{\Gamma}$ , the project is not funded when the government can extort the project's returns but it would be funded otherwise.

Finally, when the return of the project is high enough, i.e.,  $\bar{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right)$ , the project is undertaken even in the presence of government moral hazard. However, in this case, government guarantees are needed for

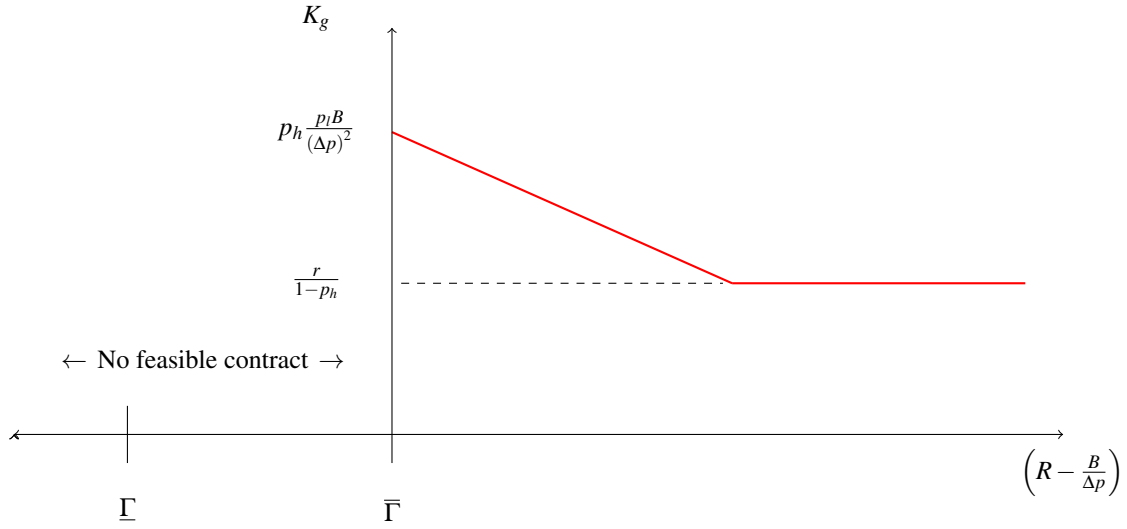


Figure 2: Guarantee in the benchmark model as a function of the relative severity of the moral hazard

the project to be funded, i.e.,  $K_g > 0$ , and the scale of the project is limited. In this case, as the severity of the moral hazard increases, higher guarantees are needed to incentivize the government not to extort the whole return of the project. Moreover, as the project's payoff increases, the government's payoff from not extorting increases and it can provide lower guarantees while still satisfying its incentive compatibility constraint. However, there is a lower bound on the government guarantee given by the outside option of the investors, which determines the maximal scale of the project in the presence of double moral hazard.

Figure 2 depicts the optimal guarantee as a function of the project payoff relative to the severity of the moral hazard  $\left(R - \frac{B}{\Delta p}\right)$ . For all figures we assume that  $\underline{\Gamma} < \bar{\Gamma}$ , which implies that, for some parameterizations, the double moral hazard is strong enough to prevent the project from being undertaken.

### 3.3 Discussion of Assumptions

There are three assumptions of our model that are worth discussing further: the government's commitment technology, the absence of government guarantees for the private sector in the event of the project's failure, and the inability of the government to invest directly.

**Government's commitment technology.** In our model, the government can commit to a contract provided it satisfies its ex-ante incentive compatibility and individual rationality constraints. Implicitly, we assume that there are big costs for the government from defaulting on the contract. These costs can be reputational, if one thinks of infrastructure projects being needed dynamically, or monetary, for example from having lower revenues due to the private sector exerting little effort to maintain the infrastructure project

properly once it anticipates that the government will renege on the contract.

**Guarantees to the private sector.** In our analysis we set the payoff to the private sector in the event of project failure to zero. Suppose we allow the government to give a guarantee  $K_p$  to the private sector in the event of project failure. Since all guarantees are funded by the government's fiscal resources  $\bar{K}$ , offering a guarantee to the private sector implies that there are fewer resources left to offer a guarantee to the investors, leading to a smaller project scale. Moreover, a guarantee to the private sector exacerbates its moral hazard problem and requires a higher payoff to the private sector when the project succeeds to induce high effort. Therefore, the optimal contract sets  $K_p = 0$  and our assumption is without loss of generality.

**Direct government investment.** Given our assumptions on the timing of the availability of resources, only the private investors can finance the infrastructure project. However, many infrastructure projects feature co-investment between the government and private investors or are even fully financed by the government. We can modify our model to consider direct government investment by allowing the government to use its fiscal resources  $\bar{K}$  to provide guarantees  $K_g I$  to the private sector or to invest  $I_g$  in the project directly at an opportunity cost  $r_g > 1$ . The extortion incentives for the government remain the same as before:  $R_b = 0$  and  $R_g$  will be as large as possible while inducing high effort by the private sector. The main differences between the benchmark model and the model with the possibility of government investment are 1) the total investment in the project is now given by  $I_g + I$ ; and 2) the fiscal constraint is now given by  $K_g I + r_g I_g \leq \bar{K}$ .

Whether the government invests in the project directly depends on the value of the optimal guarantee and the relative return of the project relative to the severity of the moral hazard. Given the linearity of the problem, in this setup the project is either fully privately financed or fully government financed.

While the possibility of direct government investment is interesting, the interaction between the government's moral hazard and the private sector's moral hazard is absent without private investment. Since double moral hazard appears to be pervasive in infrastructure projects, we abstract from direct government investment in the remainder of the paper and maintain the assumptions of the benchmark model.

## 4 Externalities, Development Rights and Tax Subsidies

We now extend our benchmark model to include other salient features of infrastructure finance. In particular, we consider externalities, development rights, and tax subsidies.

### 4.1 Externalities

Most infrastructure projects generate payoffs that go beyond the cash flows of the project. These spillovers may be valuable to the government or to the parties involved in the development and operation of the project. First, we analyze the case in which the externalities accrue to the government and not to the private parties.

We find that the higher the value of the externalities, the more the government values the project's success and thus is more willing to provide guarantees and not extort.

Formally, we model the externalities from the infrastructure project as a payoff  $XI$  that accrues to the government only if the project succeeds. The payoffs to the investors and the private sector remain unchanged from the benchmark model. However, the externalities change the government's participation and incentive compatibility constraints as follows.

The incentive compatibility constraint of the government now takes the form:

$$p_h(\bar{R}_g + X)I - (1 - p_h)K_gI \geq p_l(R + X)I - (1 - p_l)K_gI \quad (\text{ICG-E})$$

or, using that  $\bar{R}_g = \left(R - \frac{B}{\Delta p}\right)$ ,

$$K_g \geq \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) - X.$$

The externalities increase the payoff if the government succeeds, and therefore, reduce the government's incentives to extort. In the equation above, this can be seen as lower level of guarantees being sufficient to satisfy the government's incentive compatibility constraint in the presence of externalities.

The participation constraint of the government also incorporates the externalities as follows.

$$K_g \leq \frac{p_h}{1 - p_h} \left[ \left(R - \frac{B}{\Delta p}\right) + X \right]. \quad (\text{IRG-E})$$

The increase in the government's expected payoff due to the externalities makes the government more willing to provide guarantees. To summarize,

**Proposition 2. (*Externalities*)** *Relative to the benchmark model in Section 3, externalities ( $X > 0$ ) reduce the inefficiencies imposed by the double moral hazard by (weakly) increasing the scale of the project and the parameter space in which the project is financed,*

- a. If  $\left(R - \frac{B}{\Delta p}\right) < \underline{\Gamma} - X$ , the project is not funded even in the absence of any government moral hazard;*
- b. If  $\underline{\Gamma} - X \leq \left(R - \frac{B}{\Delta p}\right) < \bar{\Gamma} - X$ , the project is not funded in the presence of government moral hazard;*  
*and,*
- c. Otherwise, the infrastructure project is funded and the optimal guarantee (scale) is decreasing (increasing) in the value of the externalities  $X$ .*

Proposition 2 characterizes the optimal guarantee in the presence of externalities. As in the benchmark models, the project is not funded when the payoff of the project is too low, even in the absence of moral hazard from the government. However, when there are externalities, the parameter space in which the project is not undertaken is reduced. Moreover, since the government gets all but  $\frac{B}{\Delta p}$  from the project's cash flows,



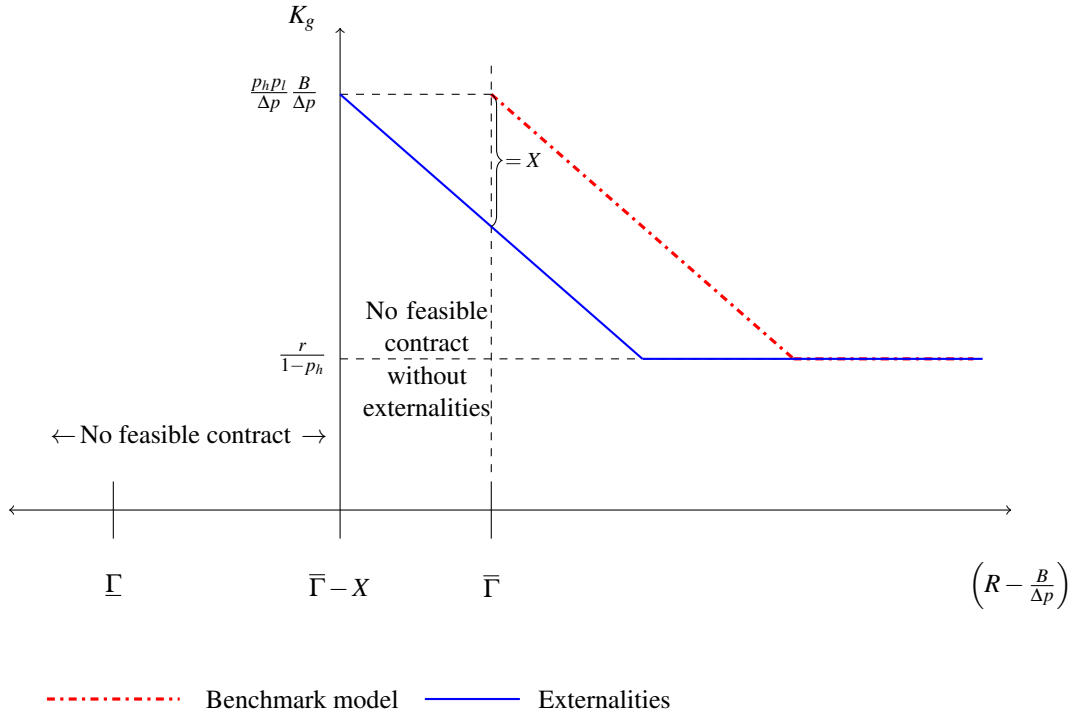


Figure 3: Optimal government guarantees under the models with and without externalities.

externalities effectively work as an increase in the return of the project and hence also increase the scale of the project. Figure 3 shows the optimal guarantees with and without externalities. The blue, solid line shows the optimal government guarantees in the model with externalities. The red, dash-dotted line shows the optimal government guarantees in the benchmark model.

For the remainder of the paper we will set the externalities to zero unless stated otherwise. However, varying the return of the project  $R$  is equivalent to changing the value of the externalities  $X$ .

## 4.2 Development Rights

Suppose now that the infrastructure project will generate other profitable ventures if it is successful (for example Second Avenue subway, and Hong Kong and Denver metros increasing values of real estate, as mentioned in Section 2). If the rights to develop these ventures can be distributed among the investors and the private sector, they can decrease the inefficiencies imposed by the double moral hazard. In the analysis that follows, we assume for simplicity that the government cannot extort the development rights and that the extra payoff can only be enjoyed by the investors and the private sector. The presence of development rights and its distribution among the private parties will affect the individual rationality constraint of the investors

and the incentive compatibility constraint of the private sector.

Formally, we model the development rights as an additional payoff  $\hat{R}I$  with  $\hat{R} > 0$  that is accrued if the project is successful. We denote by  $\hat{R}_b$  the portion of the per unit payoff  $\hat{R}$  assigned to the investors. The residual value of the development rights,  $(\hat{R} - \hat{R}_b)$ , is assigned to the private sector. While the government cannot enjoy the payoffs from the development rights directly, it can indirectly increase its payoff from not extorting  $(\hat{R} - \hat{R}_b)$  as long as  $\frac{B}{\Delta p} > (\hat{R} - \hat{R}_b)$ . Intuitively, for every dollar that the private sector receives from the development rights the government can extort an additional dollar from the project from the private sector while still inducing high effort. Hence, the maximum amount that the government can extort, denoted as  $\bar{R}_g$ , will also change when there are development rights attached to the financing of the infrastructure project.

The incentive compatibility constraint for the private sector now takes the form:

$$p_h [R - (R_b + R_g) + \hat{R} - \hat{R}_b] I \geq p_l [R - (R_b + R_g) + \hat{R} - \hat{R}_b] I + BI, \quad (\text{ICP-DR})$$

or

$$\left( R - \frac{B}{\Delta p} \right) + \hat{R} - \hat{R}_b \geq \bar{R}_g.$$

The government will extort as much as it can from the private sector while inducing high effort. Since the government cannot extort the payoffs from the development rights, the maximum the government can extort while satisfying the private sector's incentive compatibility constraint is

$$\bar{R}_g = \min \left\{ \left( R - \frac{B}{\Delta p} \right) + \hat{R} - \hat{R}_b, R \right\} = \left( R - \frac{B}{\Delta p} \right) + \min \left\{ \hat{R} - \hat{R}_b, \frac{B}{\Delta p} \right\}. \quad (1)$$

When  $\hat{R} - \hat{R}_b > \frac{B}{\Delta p}$ , the government's moral hazard will not impose any constraints on infrastructure project financing, i.e.,  $\bar{R}_g = R$ , because the incentive compatibility constraint for the private sector can be met using the payoff from the development rights only.

The incentive compatibility constraint of the government remains the same as in constraint ICG where the maximum payoff that the government can get if it does not extort all the payoff from the project is now higher. Therefore, the incentive compatibility constraint of the government becomes

$$K_g \geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - \frac{p_h}{\Delta p} \min \left\{ \hat{R} - \hat{R}_b, \frac{B}{\Delta p} \right\}. \quad (\text{ICG-DR})$$

Every dollar from the development rights assigned to the private sector increases the payoff that the government receives when it doesn't extort and relaxes the government's incentive compatibility constraint.

The individual rationality constraint of the investor also changes to

$$rI \leq p_h (R_b + \hat{R}_b) I + (1 - p_h) K_g I. \quad (\text{IRP-DR})$$

Using that the government will extort all the payoff it can from the investors, i.e.,  $R_b = 0$ , the individual rationality constraint for investors above becomes

$$\frac{r - p_h \hat{R}_b}{1 - p_h} \leq K_g.$$

Any fraction from the development rights assigned to the investors is analogous to decreasing their outside option, and hence, relaxes their participation constraint and requires a lower government guarantee for the infrastructure project to be funded.

Finally, the individual rationality constraint for the government is the same as in the benchmark model using that the government's payoff if it doesn't extort is given by Eq. (1). Then, this constraint can be written as

$$K_g \leq \frac{p_h}{1 - p_h} \left( R - \frac{B}{\Delta p} + \min \left\{ \hat{R} - \hat{R}_b, \frac{B}{\Delta p} \right\} \right). \quad (\text{IRG-DR})$$

The feasibility constraint for the government guarantee remains the same as in the benchmark model and will hold with equality. Moreover, the feasibility constraint on the fraction of development rights assigned to the investors is  $\hat{R}_b \in [0, \hat{R}]$ .

In the presence of development rights, the socially optimal financing contract in the presence of development rights solves the following problem:

$$\max_{K_g \geq 0, \hat{R}_b \in [0, \hat{R}]} (p_h (R + \hat{R}) - 1) \frac{\bar{K}}{K_g}$$

subject to

$$\begin{aligned} K_g &\geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - \frac{p_h}{\Delta p} \min \left\{ \hat{R} - \hat{R}_b, \frac{B}{\Delta p} \right\}, \\ K_g &\geq \frac{r - p_h \hat{R}_b}{1 - p_h}, \text{ and} \\ K_g &\leq \frac{p_h}{1 - p_h} \left[ \left( R - \frac{B}{\Delta p} \right) + \min \left\{ \hat{R} - \hat{R}_b, \frac{B}{\Delta p} \right\} \right]. \end{aligned}$$

As in the benchmark model, the first two constraints, the incentive compatibility constraint for the government and the individual rationality constraint for the investors, impose lower bounds on the government guarantee, while the last constraint, the individual rationality constraint for the government, imposes an

upper bound on it. The optimal financing contract is characterized in the next proposition.

**Proposition 3. (Development Rights)** *Relative to the benchmark model in Section 3, development rights ( $\hat{R} > 0$ ) reduce the inefficiencies imposed by the double moral hazard by increasing the scale of the project and the parameter space in which the project is financed:*

- a. *If  $\left(R - \frac{B}{\Delta p}\right) < \underline{\Gamma} - \hat{R}$ , the project is not funded even in the absence of any government moral hazard;*
- b. *If  $\underline{\Gamma} - \hat{R} \leq \left(R - \frac{B}{\Delta p}\right) < \bar{\Gamma} - (1 - p_l) \frac{p_h}{\Delta p} \hat{R}$ , the project is not funded in the presence of government moral hazard; and,*
- c. *Otherwise, the project is funded and the optimal guarantee (scale) is decreasing (increasing) in the return of the development rights  $\hat{R}$ .*

*In particular, if  $\hat{R} > \underline{\Gamma}$  and  $\left(R - \frac{B}{\Delta p}\right)$  is high enough also, the project becomes self-financing in that no government guarantees are necessary.*

As in the benchmark model, the project is not funded when the payoff of the project is too low, even in the absence of government moral hazard. However, the presence of development rights reduces the parameter space in which the project is not undertaken and effectively works as a reduction in the outside option of the investors.

The distribution of the development rights among the investors and the private sector depends on which marginal agent is restricting the size of the project. Intuitively, when the payoff of the project is low, the government has more incentives to extort since inducing high effort from the private sector does not increase its payoff much. In this case, compensating the private sector is relatively harder than providing guarantees to the investors. By allocating all the development rights to the private sector, the government can induce high effort from the private sector while increasing its payoff from not extorting. On the other hand, when the payoff of the project is high, the government has low incentives to extort. In this case, it is relatively easy to induce high effort from the private sector and harder to provide guarantees to the investors. By allocating all the development rights to the investors, the government reduces the guarantees that it needs to offer to the investors.

Finally, whereas in the benchmark model the scale of the project was always inefficient, high enough payoffs from development rights imply the project will always be undertaken. Moreover, development rights can make the project self-financing, provided that the return of project is high enough. Figure 4 shows the optimal guarantee and development rights assigned to the investors as a function of the project payoff relative to the severity of the moral hazard problems.

The presence of development rights and of externalities have similar implications for the optimal infrastructure financing contract: both expand the set of parameters under which the project is undertaken and the scale of the project. The main difference between these models is that with externalities, the project can

never be self-financing because the government cannot commit to not extort the return from the investors and therefore investors would never participate in the project without government guarantees, which limits the scale of the infrastructure project. In the presence of development rights, investors may still participate in the project if the return of their assigned portion of the development rights is large enough. This contrasts the effectiveness of development rights and externalities in eliminating the government's moral hazard. While externalities and development rights both ameliorate the government's moral hazard problem, only large enough development rights can eliminate it. The next section makes this distinction clear.

### 4.3 Tax subsidies

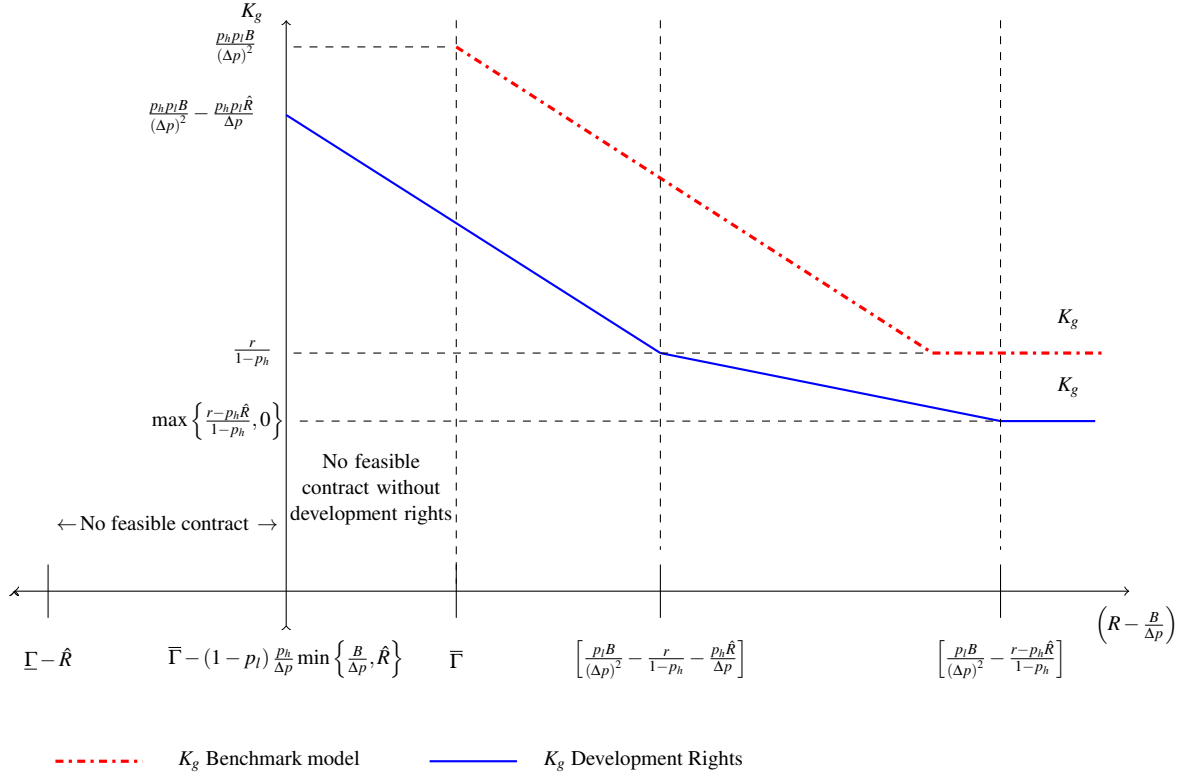
Many infrastructure financing projects involve tax subsidies to the investors or operators of the project. These tax subsidies are additional payoffs received by the private sector if the project succeeds at the expense of a lower payoff for the government. We model tax subsidies as a fraction of the externality payoffs received by the government.

Formally, we consider the same model as in Section 4.2 with the main difference that there are additional returns from the project for the government. The additional return from the project is given by  $X$ , which we interpret as the total tax revenue the government can receive from the project. The government can choose to make a fraction  $\tau$  of the return  $X$  pledgeable to the private sector either in the form of a tax subsidy to the investors or a tax abatement for the private sector. For a given value of  $\tau$ , this setup nests the model in the Section 4.2 with  $\hat{R} = \tau X$  and the return for the government is  $R_g = R + (1 - \tau)X - \frac{B}{\Delta p}$ . Proposition 4 below characterizes the optimal tax subsidy.

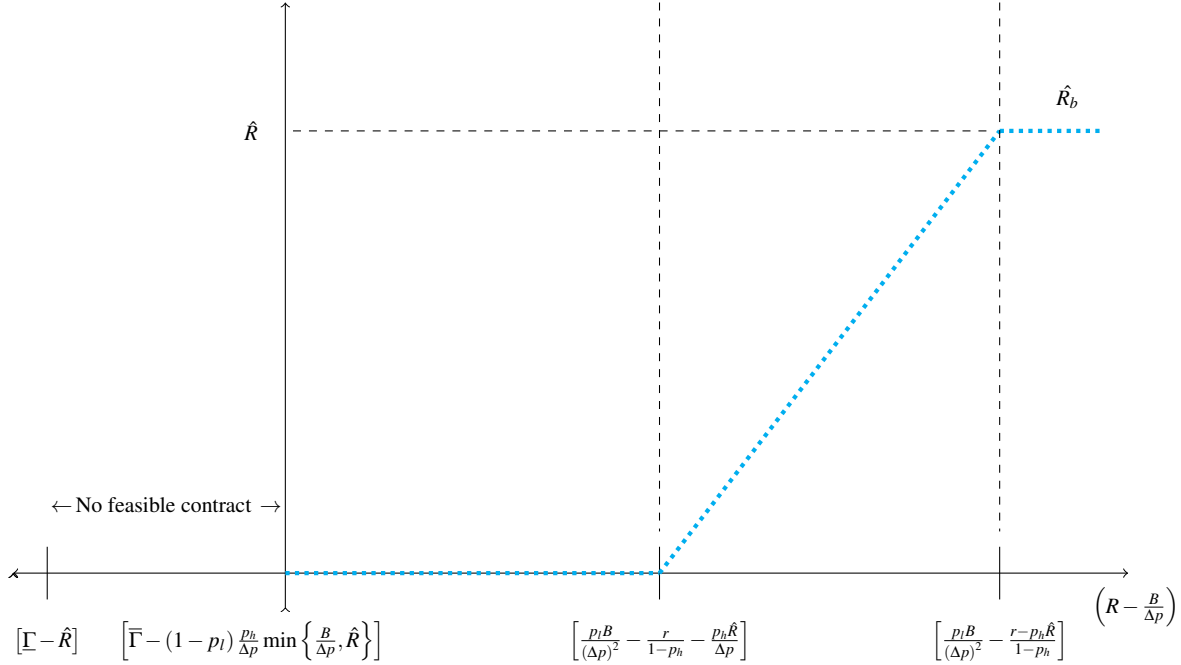
**Proposition 4. (Tax subsidies)** *The optimal financing contract with tax subsidies always requires positive tax subsidies. If  $X > \underline{\Gamma}$  and  $\left(R - \frac{B}{\Delta p}\right)$  is high enough also, the project becomes self-financing in that no government guarantees are necessary and the optimal subsidy is any  $\tau X \geq \underline{\Gamma}$ . Whenever government guarantees are necessary to undertake the project, the optimal tax subsidy is maximal and  $\tau = 1$ .*

A larger tax subsidy to the investors will reduce the government guarantee required by them to participate in the project while increasing the governments' incentives to extort. However, as we discussed above, the tax subsidy will only be fully assigned to the investors when the government's incentive to extort is weak and its incentive compatibility constraint is not binding. In this case, the optimal contract requires the subsidy to be as high as possible, which implies,  $\tau X = X$ .

When it is optimal to assign the subsidy fully to the private sector, a higher subsidy decreases the incentives of the government to extort and therefore allows for a lower government guarantee. Since the tax subsidy will be fully assigned to the private sector only when the investors' participation constraint is slack, the optimal tax subsidy is again as high as possible, i.e.,  $\tau X = X$ .



(a) Optimal government guarantees under the models with and without development rights.



(b) Distribution of development rights under the optimal contract

Figure 4: Optimal government guarantees and distribution of development rights

If the cash flows  $X$  are enough to satisfy the participation constraint of the investors without government guarantees, that is,  $X \geq \underline{\Gamma}$ , the project is always undertaken. In this case, if the return of the project is high enough, the project becomes self-financing in that government guarantees are not necessary to incentivize the government not to extort and any tax subsidy  $\tau X \geq \underline{\Gamma}$  will give investors enough incentives to fund the project.

We can interpret the non-pledgeable and pledgeable components of the additional return from the project by mapping the non-pledgeable return  $(1 - \tau)X$  to externalities (Section 4.1) the pledgeable return  $\tau X$  to development rights (Section 4.2). The main difference between these two components of the additional return from the project is that the government can commit not to extort  $\tau X$  while it cannot commit not to extort  $(1 - \tau)X$ . As the proposition above shows, it is always (weakly) optimal to make as much of the additional return pledgeable to reduce the severity of the government's moral hazard. The higher the pledgeable return, the lower the government guarantees needed for the project to be undertaken and the larger the scale of the project. Therefore, one can think of  $\hat{R}$  as development rights or as tax subsidies or abatements.

## 5 Revenue Only vs. General Obligation Financing

Governments may have access to multiple sources of cash flows to pay the investor. So far, we have implicitly considered “revenue only” (RO) financing, i.e., only the cash flows associated with the infrastructure project and the government's fiscal capacity for guarantees can be used to pay investors. However, in many instances, cash flows from other projects are also used to pay investors, for example, in “general obligation” (GO) financing. In this section, we expand the set of projects and financing contracts to allow for general obligation financing. We consider two ex-ante identical infrastructure projects and model general obligation financing as a cross-guarantee between the projects.

Formally, we consider two ex-ante identical, independent infrastructure projects,  $i = a, b$ . Each project is subject to moral hazard from the respective private sector operator. The government can choose to extort the returns of the projects after they are realized and decides whether to do so in each project independently of the what it does in the other project. To finance the projects, the government offers to investors of project  $i$  a guarantee  $K_g^i I^i$ ,  $i = a, b$  if the project fails, and an additional transfer or cross-guarantee  $K^i I^i$  from the cash flows from project  $j \neq i$  if project  $i$  fails and project  $j$  succeeds. We denote by  $\alpha \equiv \frac{I^a}{I^a + I^b}$  the fraction of total investment in project  $a$ .

**Remark** Note that since cross-guarantees can always be chosen to be zero, general obligation financing can only increase the scale of the project relative to revenue only financing (benchmark model). Hence, the analysis of interest is when general obligation financing features positive cross-guarantees, or in other

words, strictly dominates revenue only financing.

The incentive compatibility constraint of the private sector in each project  $i$  is the same as the one considered in the benchmark model, i.e.,

$$R - (R_b^i + \bar{R}_g^i) \geq \frac{B}{\Delta p}, \quad i = a, b,$$

where  $R_b^i$  and  $\bar{R}_g^i$  are, respectively, the return to the investors and the government from project  $i$  if it succeeds. As in the benchmark model, the government will extort as much as possible from the investors and it will extort all it can from the private sector while providing the private sector incentives to exert effort. Hence,  $R_b^i = 0$  and  $\bar{R}_g^i = \left(R - \frac{B}{\Delta p}\right)$  for  $i = a, b$ .

The incentive compatibility constraint of the government in each project now takes into account the expected transfers made and received from the other project. Formally,

$$\begin{aligned} p_h \bar{R}_g^j I^i - (1 - p_h) K_g^i I^i - p_h (1 - p_h) K^i I^i - p_h (1 - p_h) K^j I^j \geq \\ p_l R I^i - (1 - p_l) K_g^i I^i - p_h (1 - p_l) K^i I^i - p_l (1 - p_h) K^j I^j, \end{aligned}$$

or

$$p_h K^i - (1 - p_h) K^j \frac{1 - \alpha}{\alpha} + K_g^i \geq \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right), \quad (\text{ICG-GO})$$

for  $i \neq j$  and  $i, j = a, b$ . Cross-guarantees have two opposing effects on the government's incentives to extort. On the one hand, providing a guarantee  $K^i$  when project  $i$  fails and project  $j$  succeeds makes it more costly for the government to extort and have the private sector not exert effort in project  $i$ . This mechanism decreases the government's incentives to extort. On the other hand, providing a guarantee  $K^j$  to investors from project  $i$  to project  $j$ , (when project  $j$  fails and  $i$  succeeds) lowers the government's payoff from not extorting and, hence, tightens its incentive compatibility constraint. Which of these two effects dominates, depends on the probability of success of the projects.

The participation constraint of the investors in project  $i$  now takes into account the cross transfer from project  $j$ . It is given by

$$p_h R_b^i + (1 - p_h) K_g^i + (1 - p_h) p_h K^i \geq r \quad (\text{IRP-GO})$$

or

$$K_g^i \geq \frac{r}{1 - p_h} - p_h K^i,$$

where we used that  $R_b^i = 0$  when the government can extort.



The participation constraint of the government when cross-guarantees are allowed is

$$p_h \left( \bar{R}_g \left( I^a + I^b \right) \right) - p_h (1 - p_h) \left( K^a I^a + K^b I^b \right) - (1 - p_h) \left( K_g^a I^a + K_g^b I^b \right) \geq 0 \quad (\text{IRG-GO})$$

or

$$p_h (1 - p_h) \left( K^a \alpha + K^b (1 - \alpha) \right) + (1 - p_h) \left( K_g^a \alpha + K_g^b (1 - \alpha) \right) \leq p_h \left( R - \frac{B}{\Delta p} \right).$$

Finally, the feasibility constraints on the guarantees are given by

$$K_g^a I^a + K_g^b I^b \leq \bar{K}$$

and

$$K^i I^i \in \left[ 0, \left( R - \frac{B}{\Delta p} \right) I^j \right] \quad i, j = a, b, \quad i \neq j.$$

**Lemma.** *If both projects are undertaken, then the government guarantee is the same for both projects, i.e.,  $K_g^a = K_g^b = K_g$  and the scale of each project is  $\frac{1}{2} \frac{\bar{K}}{K_g}$ .*

Since the projects are identical and the optimal financing contract maximizes the sum of the expected payoff of the projects, it is optimal to maximize the joint scale of the projects. Since the scale of the projects is determined by the government guarantee required by the investors, it is optimal to undertake the project with the lowest required guarantee. Therefore, if both projects are undertaken it has to be the case that the government guarantees required for investors in both projects are the same.

We then obtain the following result on when cross-guarantees are desirable:

**Proposition 5. (RO vs. GO)** *Whether general obligation financing is preferred to revenue only financing depends on  $\left( R - \frac{B}{\Delta p} \right)$ , the project's return net of the moral hazard:*

- a. *If  $\left( R - \frac{B}{\Delta p} \right) < \underline{\Gamma}$ , the project is not funded even in the absence of any government moral hazard.*
- b. *If  $\underline{\Gamma} \leq \left( R - \frac{B}{\Delta p} \right) < \bar{\Gamma}$ , the project is not funded in the presence of government moral hazard.*
- c. *Otherwise, the project is funded. In this case, general obligation financing is strictly preferred to revenue only financing only if the return of the project is high enough; in other words, the optimal cross-guarantees are positive ( $K^a = K^b > 0$ ) if*

$$\left( R - \frac{B}{\Delta p} \right) > \begin{cases} \frac{\bar{\Gamma}}{(2-p_h)p_h} & \text{if } p_h > 0.5 \\ \frac{\bar{\Gamma}}{(1-p_h)} - \frac{r}{(1-p_h)} & \text{if } p_h < 0.5, \end{cases}$$

*else, revenue only financing is preferred to general obligation financing, i.e., optimal cross-guarantees are*

zero ( $K^a = K^b = 0$ ).

d. If  $\left(R - \frac{B}{\Delta p}\right) \geq \frac{1}{(1-p_h)}\Gamma$ , the project does not require any additional government guarantees, i.e.,  $K_g^a = K_g^b = 0$ , when cross-guarantees are chosen optimally.

Proposition (5) shows that general obligation financing increases the scale of the project whenever the project is undertaken. However, it does not increase the likelihood of the project being financed, at least in the symmetric case. Nevertheless, cross-guarantees can make the project self-financing if the projects payoffs are high enough.

Intuitively, cross-guarantees mainly affect the government's incentives to extort. By extorting the returns of the projects, the government cannot induce high effort from the private sector operator. Low effort by the operator in project  $a$  implies a higher probability of paying the cross guarantee from project  $b$  to project  $a$ . The increase in this probability is  $p_h(1-p_l) - p_h(1-p_h)$ , where  $p_h$  and  $p_l$  are the project's success probabilities when the private sector exerts high and low effort, respectively. At the same time, it decreases the probability with which the cross-guarantees will be paid to project  $b$  from project  $a$ . The decrease in this probability is  $p_l(1-p_h) - p_h(1-p_h)$ . When both projects are symmetric, the cross-guarantees are the same from  $a$  to  $b$  and from  $b$  to  $a$ . Therefore, the increase in the incentives to extort based on the cross-guarantees is given by

$$-\Delta p p_h + \Delta p (1 - p_h) = \Delta p (1 - 2p_h) ,$$

where  $\Delta p \equiv p_h - p_l$ . When  $p_h < \frac{1}{2}$ , cross guarantee exacerbate the moral hazard of the government and revenue only financing (setting the cross-guarantees to zero) is optimal whenever the payoff of the project is low and the incentive compatibility of the government binds. Alternatively, when  $p_h > \frac{1}{2}$ , cross-guarantees mitigate the moral hazard of the government and general obligation financing (positive cross-guarantees) are optimal as long as the return of the project is high enough to satisfy the government's participation constraint.

Finally, when the cash flow from the project is high enough, the project is self financing regardless of whether  $p_h \gtrless \frac{1}{2}$ . Any dollar pledged in the cross-guarantees cannot be extorted by the government. As a result, if the expected return of the projects is high enough, then the cross-guarantees are enough to satisfy the individual rationality constraint of the investors and no additional government guarantee is needed for the project to be undertaken.

Figures 5 show the optimal government guarantees with cross-guarantees compared with the optimal guarantees in the benchmark model without cross-guarantees. Figure 5a shows the case when  $p_h > \frac{1}{2}$  and Figure 5b shows the case when  $p_h < \frac{1}{2}$ .

In summary, revenue only financing can only be optimal when the return of the project is low, and in this case, having general obligation financing available does not affect the scale of the project. However,

when the return of the project is high, cross-guarantees create value and are positive whenever the project is implemented; in this case, general obligation financing increases the scale of the project.

As observed in Section 2, revenue only bonds account for nearly two thirds of the municipal bond market. Our model suggests that this may be the case because of a high severity of moral hazard in these projects; in these cases, providing cross-guarantees across projects in the form of general obligation bonds would lead to an increase in the inefficiency by weakening the government's incentives to participate or not to extort (conditional on participation).

## 5.1 Development Rights and General Obligation Financing

We further explore the choice between general obligation and revenue only financing in the presence of development rights. Formally, we consider the same model as in Section 5 with the addition that each project  $i$  generates an additional payoff  $\hat{R}I^i$  with  $\hat{R} > 0$  that is accrued if the project is successful, which we refer to as development rights. As in Section 4.2, this payoff can only be distributed to the private sector and cannot be extorted by the government. We denote by  $\hat{R}_b^i I^i$  the portion of the payoff from the development rights that is assigned to the investors. The residual,  $(\hat{R} - \hat{R}_b^i) I^i$ , is distributed to the private sector operator of the project. With respect to the model with general obligation financing, development rights affect the incentive compatibility constraint of the investors, which in turn changes the maximum amount that the government can extort from the private sector while still inducing high effort; development rights also affect the individual rationality constraint of the investors.

The incentive compatibility constraint of the private sector operating project  $i$  now takes the form:

$$p_h [R - (R_b^i + R_g^i) + \hat{R} - \hat{R}_b^i] I \geq p_l [R - (R_b^i + R_g^i) + \hat{R} - \hat{R}_b^i] I + BI, \quad (\text{ICP-GO-DR})$$

or, using that  $R_b^i = 0$ ,

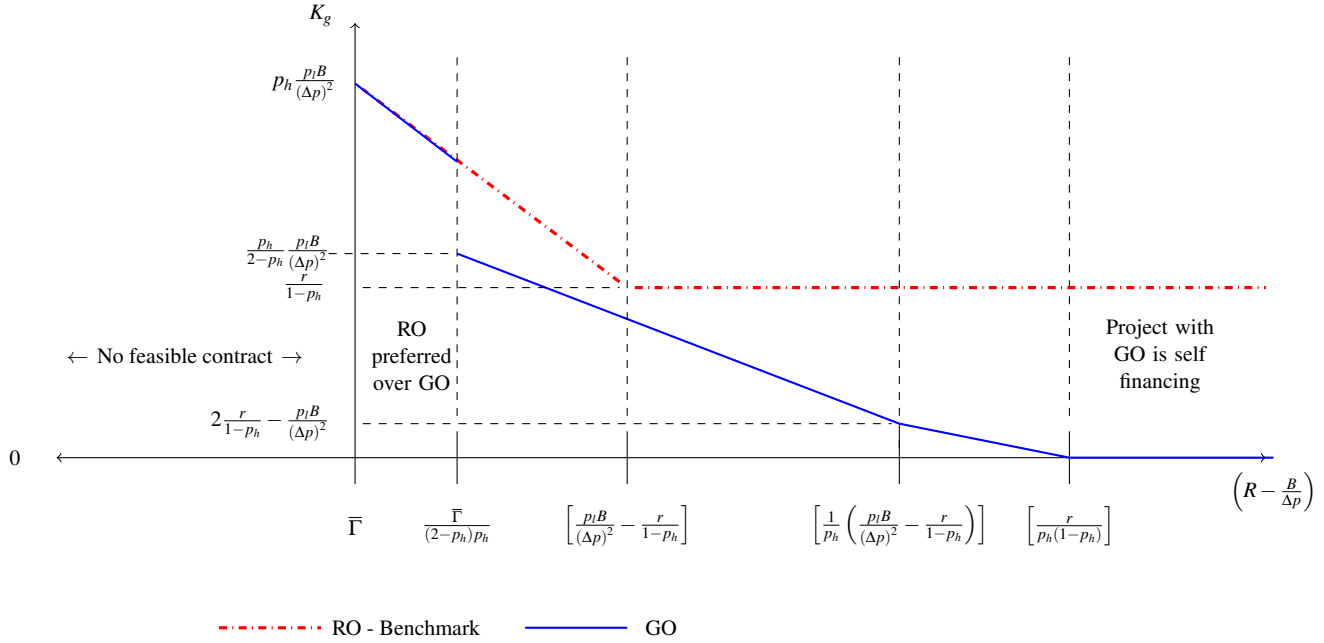
$$\left( R - \frac{B}{\Delta p} \right) + \hat{R} - \hat{R}_b^i \geq \bar{R}_g^i.$$

Since the government cannot extort any proceeds from the development rights, the maximum payoff for the government from project  $i$  is

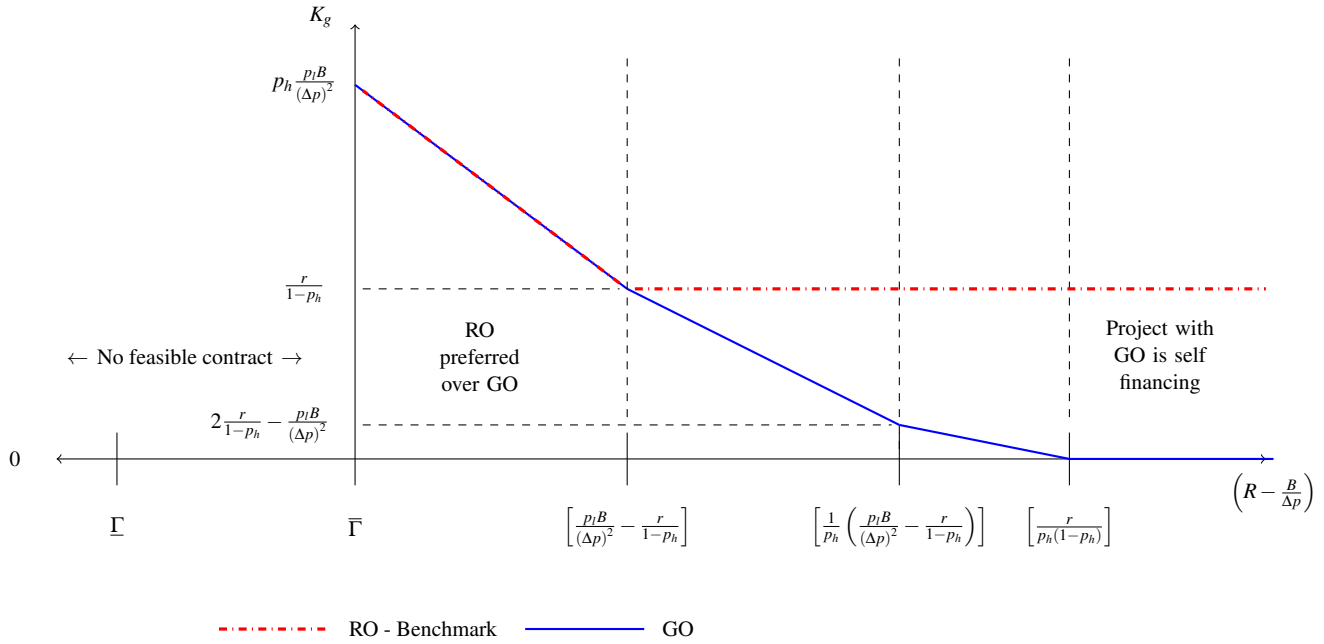
$$\bar{R}_g^i = \left( R - \frac{B}{\Delta p} \right) + \min \left\{ \hat{R} - \hat{R}_b^i, \frac{B}{\Delta p} \right\}.$$

The individual rationality constraint of the investors in project  $i$  now becomes

$$p_h (R_b^i + \hat{R}_b^i) + (1 - p_h) K_g^i + (1 - p_h) p_h K^i \geq r \quad (\text{IRP-GO-DR})$$



(a) Optimal government guarantees under cross-guarantees (GO) and under the benchmark (RO) model when  $p_h > 0.5$ .



(b) Optimal government guarantees under cross-guarantees (GO) and under the benchmark (RO) model when  $p_h < 0.5$ .

Figure 5: Optimal government guarantees when there are cross-guarantees available

or, using that  $R_b^i = 0$ ,

$$K_g^i \geq \frac{r - p_h \hat{R}_b^i}{1 - p_h} - p_h K^i.$$

We then obtain the following result on how development rights affect infrastructure financing in the presence of cross-guarantees.

**Proposition 6. (RO vs. GO with Development Rights)** *Development rights ( $\hat{R} > 0$ ) reduce the inefficiencies imposed by double moral hazard, i.e., they increase the scale of the project and the parameter space in which the project is financed, even in the presence of cross-guarantees. General obligation financing is strictly preferred to revenue only financing only if*

$$\left( R - \frac{B}{\Delta p} \right) > \begin{cases} \left( \bar{\Gamma} - (1 - p_l) \frac{p_h \hat{R}}{\Delta p} \right) \frac{1}{(2 - p_h) p_h} & \text{if } p_h > 0.5 \\ \frac{\bar{\Gamma}}{(1 - p_h)} - \frac{r}{(1 - p_h)} - \frac{p_h \hat{R}}{\Delta p} & \text{if } p_h < 0.5, \end{cases}$$

*The parameter region over which general obligation financing is strictly preferred is increasing in the value of the development rights,  $\hat{R}$ .*

Proposition 6 shows that, as in the model in Section 4.2, development rights decrease the inefficiencies imposed by the double moral hazard by increasing the scale of the project and expanding the set of parameters under which the projects are undertaken. Moreover, their presence can affect whether infrastructure financing involves positive cross-guarantees. In particular, development rights increase the parameter region over which projects are financed with general obligation financing (positive cross-guarantees). Intuitively, development rights ameliorate the government's moral hazard problem on each project, and in turn, increase the size of the cross-guarantees that can be provided.

## 6 Early-stage Government Moral Hazard

Finally, we extend the benchmark model to consider infrastructure projects with multiple stages. In the first “early” stage, the project requires government “input”, which can represent project approval, land acquisition, clearance of existing properties on the land, provision of public utilities, etc. In the second stage, once the project has gone past the government input stage, the private sector can shape the quality of the project based on its own inputs and once the project's cash flows are realized, the government can extort them, as in the benchmark model.

In this case, there are two instances in which the project can fail, after the government input stage or after the private sector input stage. The government may offer guarantees to the investors in the event the project fails after each of these instances. These guarantees expose the government to the risk of project failure and potentially ameliorate the government moral hazard problems in the two stages.

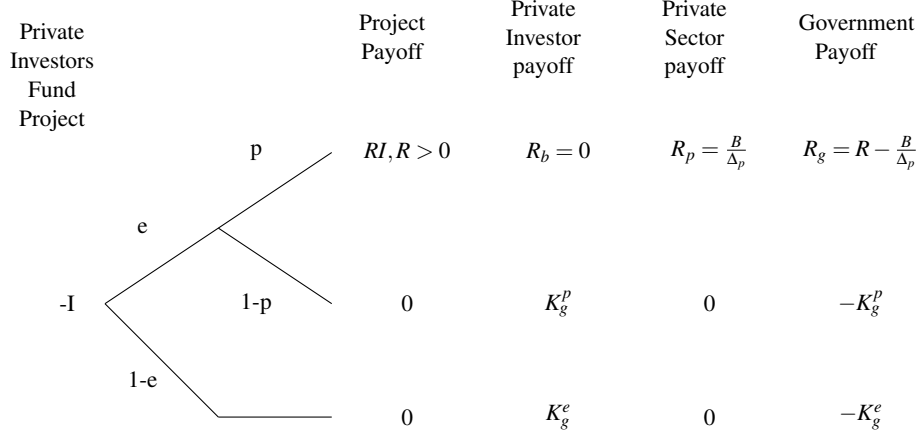


Figure 6: State space of possible outcomes and corresponding payoffs for various economic agents under the model with government moral hazard in the first stage.

Formally, in the first stage, the government can affect the probability  $e$  of the project's success through its input. If the government input is high, the project succeeds with probability  $e_h \in (0, 1)$ , else it succeeds with probability  $e_l$ ,  $0 < e_l < e_h$ . We denote as  $\Delta e$  the difference in these probabilities, that is,  $\Delta e \equiv (e_h - e_l)$ . If the government does not provide the high input, the associated officials are assumed to derive a non-pecuniary private benefit of  $bI$  with  $b > 0$ . In case the project fails in the first stage, it has no further chance of success and its payoff is zero. If the first stage of the project does not fail, the model is exactly the same as the benchmark model. The private sector can affect the probability of success of the project by exerting effort and the government can extort the project's cash flows once they are realized at the end of the second stage.

We denote by  $K_g^e I$  and  $K_g^p I$  the government guarantees if the project fails after the first and the second stages, respectively. As before, the size of these guarantees is constrained by the fiscal capacity of the government, which we take to be fixed at  $\bar{K}$ . Since either the first-stage guarantee or the second-stage guarantee is paid but not both, the fiscal constraint is

$$\max \{K_g^e, K_g^p\} I \leq \bar{K}. \quad (\text{Fiscal-constraint-GI})$$

The state space of outcomes for the projects, and project payoffs as well as payoffs to various parties (the private sector operator, the private investors, and the government) are summarized in Figure 6.

The presence of the first-stage government moral hazard reduces the parameter space in which the project is financed. Moreover, the scale of the project depends on the highest guarantee offered by the government, which in turn depends on the relative severity of the government's moral hazard in the two stages. When the government's moral hazard in the first stage, measured by  $\frac{b}{p_h \Delta e}$ , is more (less) severe than

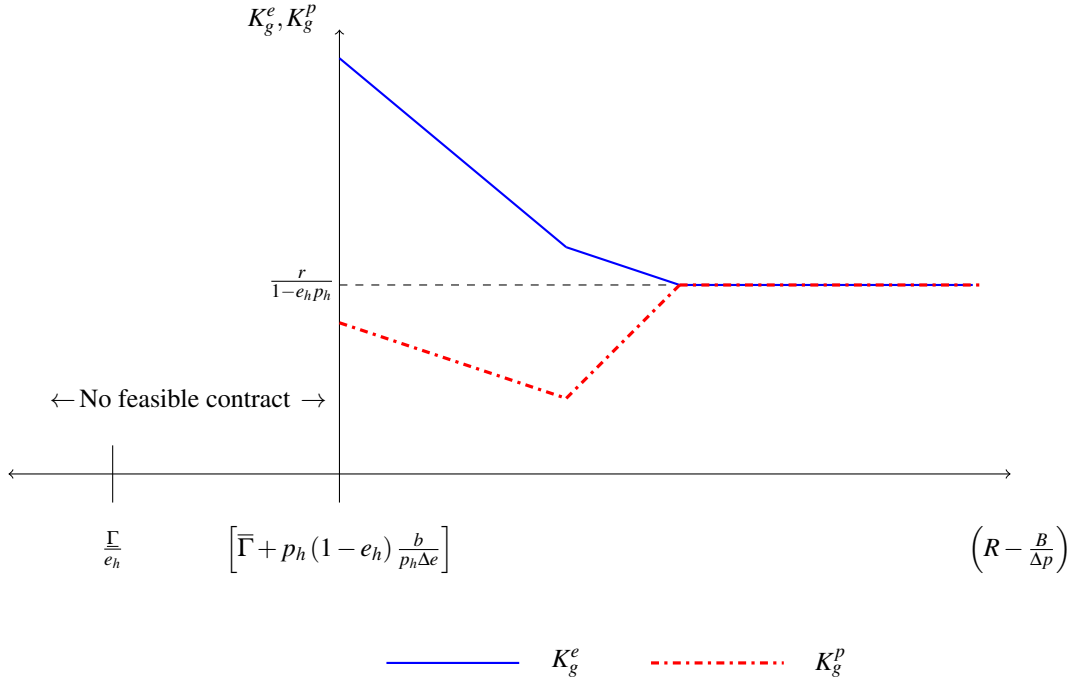
that in the second stage, measured by  $\frac{pB}{(\Delta p)^2}$ , it is harder (easier) to incentivize the government to provide high input than to incentivize it not to extort. In this case, the punishment for failing earlier needs to be higher (lower) than the one for failing in the second stage which is achieved by having the first-stage guarantee be larger (smaller) than the second-stage one. When the return of the project is high enough, the government gets a high enough return to incentivize it to provide high input and not to extort, and the government guarantees are determined by the participation constraint of the investors. In this case, the guarantees in the first and second stage are equal to maximize the scale of the project while satisfying the investors' participation constraint. The Online Appendix formally characterizes the optimal contract and the optimal guarantee structure.

Figure 7 shows the optimal government guarantees. Panels a and b respectively show the optimal guarantees for the cases in which the moral hazard of the government in the first stage is more severe than its moral hazard in the second stage and vice versa.

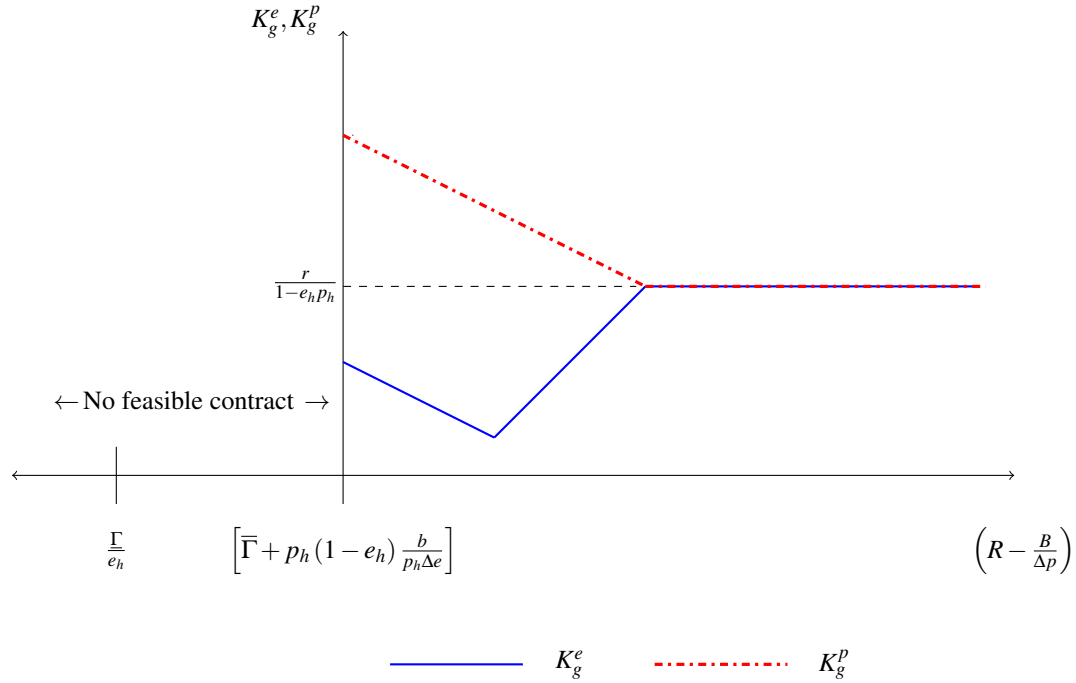
## 7 Conclusion

We analyzed the optimal design of infrastructure financing in the presence of a double moral hazard problem, viz., private sector firms need incentives to exert effort to implement and maintain projects well and governments with the ability to extort cash flows from projects once they are implemented need incentives to commit to sharing the projects' returns with the private sector (for instance, by not restricting the user fees). This double moral hazard problem limits the willingness of outside investors to fund infrastructure projects. The optimal design of infrastructure finance can ameliorate these two moral hazards using (I) government guarantees to investors against project failure; (II) bundling of development rights for the private sector and investors; (III) tax subsidies to the private parties out of infrastructure externalities; and, (IV) "general obligation" financing in the form of cross-guarantees between high-quality projects and "revenue only" financing without cross-guarantees for low-quality projects. All of these features can be found in the practice of infrastructure financing, highlighting the relevance of the double moral hazard we considered in such projects.

There are other features observed in infrastructure financing that are worthy of further investigation in a setup such as ours. A notable one is the feature of *co-investing* by governments which we briefly discussed in Section 3. There are examples of governments co-investing with the private sector to help the infrastructure projects achieve the closure of their initial financing. Consider, for instance, the provision of secured direct loans by the United States government under the Transportation Infrastructure Finance and Innovation Act (TIFIA) of 1998. In the United Kingdom too, the Treasury has established since 2009 a unit that co-lends along with private sector lenders to fund privately financed infrastructure initiatives, the stated



(a) Case 1: Government guarantee structure when first-stage moral hazard is more severe than the second-stage moral hazard, i.e.,  $\frac{b}{p_h \Delta e} \geq \frac{p_l B}{(\Delta p)^2}$ .



(b) Government guarantee structure when second-stage moral hazard is more severe than the first-stage moral hazard, i.e.,  $\frac{b}{p_h \Delta e} \leq \frac{p_l B}{(\Delta p)^2}$ .

Figure 7: Optimal guarantee structure as a function of  $\left( R - \frac{B}{\Delta p} \right)$ .



goal being to exit the investment by selling the loans in the private capital markets once the projects become self-sustaining. The Australian government also has co-lending facilities, whereby it lends on commercial terms along with private sector banks to fill the funding gap in infrastructure projects. In which situations co-investing arises as being optimal for the government in addition to providing guarantees is an important area for future research.

Finally, a natural application of our model is the design of optimal financing of health services, including ongoing issues at the time of writing such as the provision of pandemic relief and the development of vaccines, where public good aspects may induce governments to cap user fees, potentially discouraging private investment. Take, for example, the development of COVID-19 vaccines. Many governments have chosen to invest in the development of these vaccines in exchange for a guaranteed supply of vaccines at a capped fee if the vaccine is successful. These arrangements have two interesting angles. First, and in line with our model, the government's initial investment can be interpreted as a guarantee if the project is unsuccessful and as a pre-payment for the vaccines if the project succeeds. Second, the government's subsidy to the initial investment resembles co-investing in the development of vaccines. However, the government does not share in the profits of the successful vaccine directly but through the option to purchase vaccines at a favorable price. Even though neither guarantees or co-investment fully describe the private-public partnerships around COVID-19 vaccines, both tools seem important in achieving the financing of the vaccine development. The exact mechanism through which this is achieved is worth studying.

## References

- Alm, James, (2010), "Municipal Finance of Urban Infrastructure: Knowns and Unknowns," The Brookings Global Economy and Development working paper series, May.
- Andonov, Aleksandar, Roman Kräussl and Joshua D. Rauh, "The Subsidy to Infrastructure as an Asset Class," Working paper, SSRN, (2020).
- Ang, Andrew, Vineer Bhansali and Yuhang Xing, "Taxes on Tax-Exempt Bonds", The Journal of Finance , APRIL 2010, Vol. 65, No. 2 (APRIL 2010), pp. 565-601
- Bolton, Patrick and Mathias Dewatripoint, "Contract Theory", MIT Press, 2004.
- Cestau Dario Hollifield Burton, Li Dan, and Schürhoff Norman, "Municipal Bond Markets," Annual Review of Financial Economics Vol. 11:65-84 (Volume publication date December 2019)
- "Public and private financing of infrastructure Policy challenges in mobilizing finance", EIB Papers Volume 15 No 2, (2010).

- Banerjee, Shantanu, Ufuk Güçbilmez, Grzegorz Pawlina, “Optimal exercise of jointly held real options: A Nash bargaining approach with value diversion”, *European Journal of Operational Research*, Vol. 239, no. 2 (2014), pp. 565-578,
- Alexander W. Butler, Larry Fauver and Sandra Mortal, “Corruption, Political Connections, and Municipal Finance,” *The Review of Financial Studies* , Jul., 2009, Vol. 22, No. 7 (Jul., 2009), pp. 2873- 2905.
- Cooper, Russell, and Thomas W. Ross. ”Product Warranties and Double Moral Hazard.” *The RAND Journal of Economics* 16, no. 1 (1985): 103-13.
- Eswaran, Mukesh, and Ashok Kotwal. ”A Theory of Contractual Structure in Agriculture.” *The American Economic Review* 75, no. 3 (1985): 352-67.
- Edward M. Gramlich, “Infrastructure Investment: A Review Essay,” *Journal of Economic Literature*, Vol. 32, No. 3 (Sep., 1994), pp. 1176 - 1196.
- Green, Richard C, 1993, A simple model of the taxable and tax-exempt yield curves, *Review of Financial Studies* 6, pp. 233-264.
- Gupta, Arpit, Stijn Van Nieuwerburgh and Constantine Kontokosta, “Take the Q Train: Value Capture of Public Infrastructure Projects,” (2020), Working paper SSRN
- “Infrastructure Partnerships Australia: Financing Infrastructure in the Global Financial Crisis,” (2009), March.
- Casamatta, Catherine, “Financing and Advising: Optimal Financial Contracts with Venture Capitalists,” *Journal of Finance*, Volume 58, Issue 5, October 2003, pp. 2059-2085.
- Hellmann, Thomas F. and Puri, Manju, *Venture Capital and the Professionalization of Start-Up Firms: Empirical Evidence* (September 2000). AFA 2001 New Orleans; Sauder School of Business Working Paper. Available at SSRN: <https://ssrn.com/abstract=243149> or <http://dx.doi.org/10.2139/ssrn.243149>.
- Lewis, Gregory and Patrick Bajari, (2014), “Moral Hazard, Incentive Contracts, and Risk: Evidence from Procurement,” *The Review of Economic Studies*, Vol. 81, No. 3 (288) (July 2014), pp. 1201-1228.
- Liu Cheol and John L. Mikesell, “The Impact of Public Officials’ Corruption on the Size and Allocation of U.S. State Spending,” (2014), *Public Administration Review*.
- Longstaff, Francis, “Municipal Debt and Marginal Tax Rates: Is There a Tax Premium in Asset Prices?,” *The Journal of Finance* , JUNE 2011, Vol. 66, No. 3 (JUNE 2011), pp. 721-751

David Martimort and Wilfried Sand-Zantman, (2006), "Signalling and the design of delegated management contracts for public utilities," *RAND Journal of Economics* Vol. 37, No. 4, Winter 2006, pp. 763 - 782.

Francesca Medda, (2007), "A game theory approach for the allocation of risks in transport public private partnerships," *International Journal of Project Management* 25 (2007), pp. 213-228.

Perotti, E.C. "Credible Privatization," *American Economic Review*, Vol. 85 (1995), pp. 847-859.

Repullo, Rafael and Suarez, Javier, *Venture Capital Finance: A Security Design Approach*. CEMFI Working Paper No. 9804. Available at SSRN: <https://ssrn.com/abstract=145134> or <http://dx.doi.org/10.2139/ssrn.145134>

Romano, Richard E. "Double Moral Hazard and Resale Price Maintenance." *The RAND Journal of Economics* 25, no. 3 (1994), pp. 455-66.

Tirole, Jean. "The Theory of Corporate Finance". Princeton Univeristy Press. 1992.

"Transportation Infrastructure Finance and Innovation Act, (2011)," Tax Payers for common sense, making governments work.

# APPENDIX

## A Benchmark Model

In this section, we characterize the optimal contracts for the benchmark model and the model with externalities presented in Section (3), and we provide the proofs for the propositions in this section.

### Characterization of optimal contract in the benchmark model

The optimal guarantee solves

$$\max_{K_g \geq 0} (p_h R - 1) \frac{\bar{K}}{K_g}$$

subject to

$$K_g \geq \frac{r}{1 - p_h} \tag{IRP}$$

$$(1 - p_h) K_g \leq p_h \left( R - \frac{B}{\Delta p} \right), \quad \text{and} \tag{IRG}$$

$$K_g \geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right), \tag{ICG}$$

where Eq. (IRP) is the individual rationality constraint of the investors, Eq. (IRG) is the individual rationality of the government, and Eq. (ICG) is the incentive compatibility constraint of the government. Note that in order to satisfy both individual rationality constraints in Eqs. (IRP) and (IRG) it must be the case that

$$\frac{r}{p_h} \leq \left( R - \frac{B}{\Delta p} \right). \tag{1}$$

Since the scale of the project and hence the objective function are decreasing in the government guarantee the optimal financing contract of the infrastructure project will have the lowest guarantee that satisfies the three constraints above. Moreover, the individual rationality constraint of the investors and the incentive compatibility constraint of the government impose lower bounds on the government guarantee. Therefore, there are two possible cases depending on which constraint binds.

If the individual rationality constraint of the investors binds, we have  $K_g = \frac{r}{1 - p_h}$ . In this case, the incentive compatibility constraint of the government will be satisfied as long as

$$\frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} \leq \left( R - \frac{B}{\Delta p} \right).$$

If the incentive compatibility constraint of the government binds, we have

$$K_g = \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right).$$

In this case, the individual rationality constraint of the investors will be satisfied as long as

$$\frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} \geq \left( R - \frac{B}{\Delta p} \right),$$

and the individual rationality constraint of the government will be satisfied if

$$(1 - p_h) \frac{p_l B}{(\Delta p)^2} \leq \left( R - \frac{B}{\Delta p} \right).$$

Finally, the project will be financed as long as

$$\max \left\{ \frac{r}{p_h}, (1 - p_h) \frac{p_l B}{(\Delta p)^2} \right\} \leq \left( R - \frac{B}{\Delta p} \right). \quad (2)$$

### Proof of Proposition 1 (Inefficiency of double moral hazard)

a) **(Feasibility)** Define  $\underline{\Gamma} \equiv \frac{r}{p_h}$  and  $\bar{\Gamma} \equiv (1 - p_h) \frac{p_l B}{(\Delta p)^2}$ . Then, since Eq. (1) has to be satisfied for the participation constraints of the investors and the government to be satisfied, the contract is not funded even in the absence of moral hazard if  $\left( R - \frac{B}{\Delta p} \right) < \underline{\Gamma}$ , which proves the statement in part i. of the proposition.

Using the definitions of  $\underline{\Gamma}$  and  $\bar{\Gamma}$  and Eq. (2) in the analysis above, we have that, in the presence of moral hazard, the project will be financed as long as  $\max \{ \underline{\Gamma}, \bar{\Gamma} \} \leq \left( R - \frac{B}{\Delta p} \right)$ . Hence, using part a) i., it follows that if  $\underline{\Gamma} \leq \left( R - \frac{B}{\Delta p} \right) < \bar{\Gamma}$  the project is not funded in the presence of moral hazard. These two statements prove parts ii. and iii. of the proposition.

b) **(Scale)** From the characterization of the optimal contract in the section above, it follows that, if the project is funded, the optimal government guarantee is given by

$$K_g = \max \left\{ \frac{r}{1 - p_h}, \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) \right\},$$

which is always positive and weakly decreasing in  $\left( R - \frac{B}{\Delta p} \right)$ . This proves part b) of the proposition. □

### Characterization of optimal contract with externalities

When there are externalities associated with the project, the optimal guarantee solves

$$\max_{K_g \geq 0} (p_h (R + X) - 1) \frac{\bar{K}}{K_g}$$

subject to

$$K_g \geq \frac{r}{1 - p_h}, \quad (\text{IRP-E})$$

$$K_g \leq \frac{p_h}{1 - p_h} \left[ \left( R - \frac{B}{\Delta p} \right) + X \right], \text{ and} \quad (\text{IRG-E})$$

$$K_g \geq \frac{p_l B}{(\Delta p)^2} - \left[ \left( R - \frac{B}{\Delta p} \right) + X \right], \quad (\text{ICG-E})$$

where we used the fact that  $R_b = 0$  and that the fiscal constraint of the government holds with equality. Note that for there to be a contract that satisfies both individual rationality constraints, it has to be the case that

$$\frac{r}{p_h} - X \leq \left( R - \frac{B}{\Delta p} \right). \quad (3)$$

Since the objective function is maximized when  $K_g$  is minimized, if there is a feasible contract, one of the three lower bounds for  $K_g$  have to be satisfied with equality. Therefore,

$$K_g = \max \left\{ 0, \frac{r}{1-p_h}, \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - X \right\}. \quad (4)$$

When the individual rationality constraint of the investors binds,

$$K_g = \frac{r}{1-p_h}.$$

For the incentive compatibility of the government ICG-E to be satisfied, it must be the case that

$$\left( R - \frac{B}{\Delta p} \right) \geq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} - X,$$

and to satisfy the individual rationality constraint of the government IRG-E it must be that

$$\frac{r}{p_h} - X \leq \left( R - \frac{B}{\Delta p} \right).$$

When ICG-E binds,

$$K_g = \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - X.$$

To satisfy IRP-E and IRG-E we must have

$$(1-p_h) \frac{p_l B}{(\Delta p)^2} - X \leq \left( R - \frac{B}{\Delta p} \right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} - X.$$

Finally, if

$$\left( R - \frac{B}{\Delta p} \right) \leq \max \left\{ \frac{r}{p_h} - X, (1-p_h) \frac{p_l B}{(\Delta p)^2} - X \right\} \quad (5)$$

the infrastructure project is not financed.

## Proof of Proposition 2 (Externalities)

a) Using that  $\underline{\Gamma} = \frac{r}{p_h}$  and Eq. (3) we have that if  $\left( R - \frac{B}{\Delta p} \right) < \underline{\Gamma} - X$  there is no contract that satisfies the incentive compatibility constraints of the government and the investors and, therefore, the project is not funded even in the absence of moral hazard.

b) From Eq. (5) and using that  $\bar{\Gamma} = (1-p_h) \frac{p_l B}{(\Delta p)^2}$  it follows that if

$$\underline{\Gamma} - X \leq \left( R - \frac{B}{\Delta p} \right) < \bar{\Gamma} - X$$

the project is not financed in the presence of moral hazard.

c) Finally, from the characterization in the section above, it follows that the project is financed if  $\max\{\underline{\Gamma}, \bar{\Gamma} - X\} \leq \left(R - \frac{B}{\Delta p}\right)$  and from Eq. (4) the optimal guarantee is decreasing, and the scale increasing, in the value of the externalities  $X$ . This proves the last part of the proposition.  $\square$

## B Development Rights

In this section we characterize the optimal contract for the models with development rights and tax subsidies presented in Section (4.2). We also provide the proofs for the propositions in this section.

### Characterization of optimal contract with development rights

When the private sector faces moral hazard and the government can extort the return of the project, the optimal infrastructure funding contract solves

$$\max_{K_g \geq 0, \hat{R}_b \in [0, \hat{R}]} (p_h (R + \hat{R}) - 1) \frac{\bar{K}}{K_g}$$

subject to

$$(1 - p_h) K_g + p_h \hat{R}_b \geq r, \quad (\text{IRP-DR})$$

$$(1 - p_h) K_g \leq p_h R_g, \quad (\text{IRG-DR})$$

$$K_g \geq \frac{p_l R - p_h R_g}{\Delta p}, \quad (\text{ICG-DR})$$

where  $R_g = R - \frac{B}{\Delta p} + \min\left\{\frac{B}{\Delta p}, \hat{R} - \hat{R}_b\right\}$ , we used the fact that  $R_b = 0$ , and that the fiscal constraint of the government binds. Since the objective function is maximized when  $K_g$  is minimized, if there is a feasible contract, one of the three lower bounds for  $K_g$  have to be satisfied with equality. Therefore,

$$K_g = \max\left\{0, \frac{r - p_h \hat{R}_b}{1 - p_h}, \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) - \frac{p_h}{\Delta p} \min\left\{\frac{B}{\Delta p}, \hat{R} - \hat{R}_b\right\}\right\}, \quad (6)$$

where we used the definition of  $R_g$  in the incentive compatibility constraint for the government.

When the individual rationality constraint of the investors binds,

$$K_g = \frac{r - p_h \hat{R}_b}{1 - p_h},$$

the scale of the project will be maximized by setting  $\hat{R}_b = \hat{R}$ . For this guarantee to be feasible, it has to satisfy the individual rationality constraint of the government, i.e.,

$$\frac{r - p_h \hat{R}}{p_h} \leq \left(R - \frac{B}{\Delta p}\right).$$

Moreover, the incentive compatibility constraint of the government will be satisfied if

$$\left(R - \frac{B}{\Delta p}\right) \geq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} + \frac{p_h \hat{R}}{1 - p_h}.$$

If  $\hat{R} > \frac{r}{p_h}$ , the development rights are enough to satisfy the investors' individual rationality constraint and no guarantees are needed to fund the project.

On the other hand, when the incentive compatibility constraint of the government binds, we have

$$K_g = \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - \frac{p_h}{\Delta p} \min \left\{ \frac{B}{\Delta p}, \hat{R} - \hat{R}_b \right\},$$

and the scale of the project is maximized by setting  $\hat{R}_b = 0$ . Note that it will never be the case that  $\frac{B}{\Delta p} \leq \hat{R} - \hat{R}_b$  while the government incentive compatibility constraint binds and the government guarantee  $K_g$  is greater than zero. Then, we can remove the min operator from the expression above. In this case, the individual rationality constraint of the investors is satisfied if

$$\left( R - \frac{B}{\Delta p} \right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} - \frac{p_h}{\Delta p} \hat{R}.$$

This constraint also implies that  $K_g > 0$ . The individual rationality constraint of the government is satisfied if

$$(1-p_h) \frac{p_l B}{(\Delta p)^2} - (1-p_l) \frac{p_h}{\Delta p} \hat{R} \leq \left( R - \frac{B}{\Delta p} \right).$$

Finally, for values of  $R - \frac{B}{\Delta p}$  such that

$$\frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} - \frac{p_h}{\Delta p} \hat{R} \leq R - \frac{B}{\Delta p} \leq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} + \frac{p_h \hat{R}}{1-p_h}$$

the individual rationality constraint of the investors and the incentive compatibility constraint of the government bind and the development rights assigned to the investors satisfy

$$\frac{r - p_h \hat{R}_b}{1-p_h} = \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - \frac{p_h}{\Delta p} (\hat{R} - \hat{R}_b).$$

In this case,  $\hat{R}_b \in [0, \hat{R}]$  and it is increasing in  $\left( R - \frac{B}{\Delta p} \right)$ .

Then, if the project is financed, the optimal guarantee is

$$K_g = \max \left\{ 0, \frac{r - p_h \hat{R}_b}{1-p_h}, \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - \frac{p_h}{\Delta p} (\hat{R} - \hat{R}_b) \right\}, \quad (7)$$

where  $\hat{R}_b$ , the return from the development rights assigned to the investor, is given by

$$\hat{R}_b = \begin{cases} 0 & \text{if } \left( R - \frac{B}{\Delta p} \right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} - \frac{p_h}{\Delta p} \hat{R} \\ \hat{R} & \text{if } \left( R - \frac{B}{\Delta p} \right) \geq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} + \frac{p_h \hat{R}}{1-p_h} \\ \hat{R}_b^* \in (0, \hat{R}) & \text{otherwise} \end{cases}, \quad (8)$$

Note that if

$$\left( R - \frac{B}{\Delta p} \right) < \max \left\{ \frac{r}{p_h} - \hat{R}, (1-p_h) \frac{p_l B}{(\Delta p)^2} - (1-p_l) \frac{p_h}{\Delta p} \hat{R} \right\} \quad (9)$$

the project is not financed.



### Proof of Proposition 3 (Development Rights)

a. Using that  $\underline{\Gamma} = \frac{r}{p_h}$  and using the participation constraints IRP–DR and IRG–DR we have that there is no contract that satisfies the individual rationality constraints of the investors and the government simultaneously if  $\left(R - \frac{B}{\Delta p}\right) < \underline{\Gamma} - \hat{R}$ . Then, in this case there is no feasible contract even in the absence of government moral hazard.

b. Using Eq. (9), the definition of  $\underline{\Gamma}$  and that  $\bar{\Gamma} = (1 - p_h) \frac{p_l B}{(\Delta p)^2}$ , we have that if

$$\underline{\Gamma} - \hat{R} \leq \left(R - \frac{B}{\Delta p}\right) < \bar{\Gamma} - (1 - p_l) \frac{p_h \hat{R}}{\Delta p}$$

the project is not financed in the presence of double moral hazard.

c. From the analysis above and Eqs. (7) and (8) we have that when  $\left(R - \frac{B}{\Delta p}\right) \geq \bar{\Gamma} - (1 - p_l) \frac{p_h \hat{R}}{\Delta p}$  the project is funded and the optimal government guarantee is decreasing in the return of the development rights. Moreover, it follows from the analysis in the previous section that if

$$\left(R - \frac{B}{\Delta p}\right) \geq \max \left\{ \underline{\Gamma} - \hat{R}, \frac{p_l B}{(\Delta p)^2} - \frac{R}{1 - p_h} + \frac{p_h \hat{R}}{1 - p_h} \right\}$$

and if  $\underline{\Gamma} \leq \hat{R}$ ,  $K_g = 0$  and the project becomes self-financing. □

### Characterization of optimal contract with tax Subsidies

When there are tax subsidies, the optimal contract solves

$$\max_{\tau \in [0, 1], \hat{R}_b \in [0, \tau X], K_g \geq 0} (p_h (R + X) - 1) \frac{\bar{K}}{K_g}$$

subject to

$$(1 - p_h) K_g + p_h \hat{R}_b \geq r, \quad (\text{IRP-TS})$$

$$(1 - p_h) K_g \leq p_h R_g, \quad (\text{IRG-TS})$$

$$K_g \geq \frac{p_l (R + (1 - \tau) X) - p_h R_g}{\Delta p}, \quad (\text{ICG-TS})$$

where  $R_g = R + (1 - \tau) X - \frac{B}{\Delta p} + \min \left\{ \frac{B}{\Delta p}, \tau X - \hat{R}_b \right\}$ , we used the fact that  $R_b = 0$ , and that the fiscal constraint of the government binds. Note that for a given value of  $\tau$ , this is the same problem solved in the previous section for development rights. Then, using the optimal government guarantees and distribution of development rights in Equations (7) and (8) we have that for a given  $\tau$  the optimal government guarantee is

$$K_g = \max \left\{ 0, \frac{r - p_h \hat{R}_b}{1 - p_h}, \frac{p_l B}{(\Delta p)^2} - \left( R + (1 - \tau) X - \frac{B}{\Delta p} \right) - \frac{p_h}{\Delta p} (\tau X - \hat{R}_b) \right\}, \quad (10)$$

where  $\hat{R}_b$ , the return from the development rights assigned to the investors, is given by

$$\hat{R}_b = \begin{cases} 0 & \text{if } \left( R - \frac{B}{\Delta p} \right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} - (1 - \tau) X - \frac{p_h}{\Delta p} \tau X \\ \tau X & \text{if } \left( R - \frac{B}{\Delta p} \right) \geq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} - (1 - \tau) X + \frac{p_h \tau X}{1 - p_h} \\ \hat{R}_b^* \in (0, \tau X) & \text{otherwise} \end{cases}, \quad (11)$$

Since the objective function is maximized when  $K_g$  is minimized, if there is a feasible contract, the optimal choice of  $\tau$  is given  $\tau = 1$  whenever  $\hat{R}_b = \{0, \tau X\}$ , and when  $\hat{R}_b \in (0, \tau X)$ , we have that  $\tau$  will be chosen to maximize  $\hat{R}_b$  such that

$$\frac{r - p_h \hat{R}_b}{1 - p_h} = \frac{p_l B}{(\Delta p)^2} - \left( R + (1 - \tau)X - \frac{B}{\Delta p} \right) - \frac{p_h}{\Delta p} (\tau X - \hat{R}_b).$$

Then,

$$\hat{R}_b = \frac{\Delta p}{(1 - p_l)} \frac{(1 - p_h)}{p_h} \left[ \frac{r}{1 - p_h} - \frac{p_l B}{(\Delta p)^2} + \left( R + X - \frac{B}{\Delta p} \right) + \frac{p_l}{\Delta p} \tau X \right],$$

which is increasing in  $\tau$ . Hence,  $\tau$  will be as high as possible, i.e.,  $\tau = 1$ .

When  $X > \frac{r}{p_h}$ , any subsidy  $\tau \geq \frac{r}{p_h X}$  will attain the optimal scale of the project and make the project self financing provided the return of the project is high enough, i.e., that

$$\left( R - \frac{B}{\Delta p} \right) \geq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} - (1 - \tau)X + \frac{p_h \tau X}{1 - p_h}.$$

### Proof of Proposition 4 (Tax subsidies)

The proof follows directly from the characterization of the equilibrium above using the definition of  $\underline{\Gamma}$  and  $\bar{\Gamma}$ . From Eqs. (10) and (11) we have that the government guarantee is decreasing in  $X$  and hence the scale of the project is increasing in  $X$ . □

## C Revenue Only vs. General Obligation Financing

In this section we characterize the optimal contract for the model in Section (5) when there are cross-guarantees and provide the proofs of the results in this section.

### Characterization of optimal contract

In this case, the government solves

$$\begin{aligned} & \max_{\alpha \in [0, 1], K_g^j \geq 0} (p_h R - 1)I \\ & K^i I^i \in \left[ 0, \left( R - \frac{B}{\Delta p} \right) I^j \right], \end{aligned}$$

s.t.

$$\frac{r}{1 - p_h} \leq p_h K^a + K_g^a, \quad (\text{IRPA-GO})$$

$$\frac{r}{1 - p_h} \leq p_h K^b + K_g^b, \quad (\text{IRPB-GO})$$

$$(1 - p_h) \left[ p_h \left( K^a \alpha + K^b (1 - \alpha) \right) + \left( K_g^a \alpha + K_g^b (1 - \alpha) \right) \right] \leq p_h \left( R - \frac{B}{\Delta p} \right), \quad (\text{IRG-GO})$$

$$p_h K^a - (1 - p_h) K^b \frac{1 - \alpha}{\alpha} + K_g^a + \left( R - \frac{B}{\Delta p} \right) \geq \frac{p_l B}{(\Delta p)^2}, \quad (\text{ICGA-GO})$$

$$p_h K^b - (1 - p_h) K^a \frac{\alpha}{1 - \alpha} + K_g^b + \left( R - \frac{B}{\Delta p} \right) \geq \frac{p_l B}{(\Delta p)^2}, \quad \text{and} \quad (\text{ICGB-GO})$$

$$\alpha I K_g^a + (1 - \alpha) I K_g^b \leq \bar{K}. \quad (\text{Fiscal-Constraint-GO})$$

Fiscal-Constraint-GO holds with equality in equilibrium. Therefore, one can rewrite the objective function as follows

$$(p_h R - 1) \frac{\bar{K}}{\alpha K_g^a + (1 - \alpha) K_g^b}.$$

**Lemma.** *If both projects are undertaken, then the government guarantee is the same for both projects, i.e.,  $K_g^a = K_g^b = K_g$  and the scale of each project is  $\frac{1}{2} \bar{K}$ .*

*Proof.* Suppose to the contrary that  $K_g^b > K_g^a$  and  $\alpha \in (0, 1)$ . Then, one could increase  $\alpha$  and increase  $K^b$  while still satisfying all the constraints and increasing the objective function. Note that an increase in  $\alpha$  would relax ICGA-GO and the upper bound on  $K^b$  while tightening ICGB-GO. But one could increase  $K^b$  to guarantee that IRPB-GO and ICGB-GO hold while still satisfying the rest of the constraints. Analogously if  $K_g^b < K_g^a$ . Hence, if both projects are undertaken, then we must have  $K_g^a = K_g^b$ .  $\square$

Using the Lemma above, the problem becomes

$$K^i I^i \in \left[ 0, \left( R - \frac{B}{\Delta p} \right) I^i \right], \quad (p_h R - 1) \frac{\bar{K}}{K_g}$$

$$\alpha \in [0, 1], K_g \geq 0$$

$$\frac{r}{1 - p_h} \leq p_h K^a + K_g, \quad (\text{IRA-GO})$$

$$\frac{r}{1 - p_h} \leq p_h K^b + K_g, \quad (\text{IRB-GO})$$

$$p_h (1 - p_h) \left( \alpha K^a + (1 - \alpha) K^b \right) + (1 - p_h) K_g \leq p_h \left( R - \frac{B}{\Delta p} \right), \quad (\text{IRG-GO})$$

$$p_h K^a - (1 - p_h) K^b \frac{1 - \alpha}{\alpha} + K_g \geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right), \text{ and} \quad (\text{ICGA-GO})$$

$$p_h K^b - (1 - p_h) K^a \frac{\alpha}{1 - \alpha} + K_g \geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right). \quad (\text{ICGB-GO})$$

To be able to satisfy the three individual rationality constraints it must be the case that

$$\frac{r}{p_h} \leq \left( R - \frac{B}{\Delta p} \right). \quad (12)$$

Moreover, the individual rationality constraints of the investor and the incentive compatibility constraints of the government impose lower bounds on the government guarantee  $K_g$  as follows:

$$K_g \geq \max \left\{ \frac{r}{1 - p_h} - p_h K^a, \frac{r}{1 - p_h} - p_h K^b, \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - \left( p_h K^a - (1 - p_h) K^b \frac{1 - \alpha}{\alpha} \right), \right. \\ \left. \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - \left( p_h K^b - (1 - p_h) K^a \frac{\alpha}{1 - \alpha} \right), 0 \right\}.$$

Note that this constraint is minimized at  $\alpha = \frac{1}{2}$ , which implies

$$K_g \geq \max \left\{ \frac{r}{1-p_h} - p_h K^a, \frac{r}{1-p_h} - p_h K^b, \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - (p_h K^a - (1-p_h) K^b), \right. \\ \left. \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - (p_h K^b - (1-p_h) K^a), 0 \right\}.$$

Moreover, the constraint is also minimized when  $K^a = K^b$ . Hence,

$$K_g \geq \max \left\{ \frac{r}{1-p_h} - p_h \hat{K}, \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - (2p_h - 1) \hat{K}, 0 \right\}, \quad (13)$$

where  $K^a = K^b = \hat{K}$  and

$$0 \leq \hat{K} \leq \left( R - \frac{B}{\Delta p} \right).$$

There are two cases, depending on whether  $p_h \geq \frac{1}{2}$ .

1) If  $p_h \geq \frac{1}{2}$ , the lower bound on  $K_g$  is always decreasing in  $\hat{K}$ . In this case, it is optimal to set

$$\hat{K} = \left( R - \frac{B}{\Delta p} \right), \quad (14)$$

and

$$K_g = \max \left\{ \frac{r}{1-p_h} - p_h \left( R - \frac{B}{\Delta p} \right), \frac{p_l B}{(\Delta p)^2} - 2p_h \left( R - \frac{B}{\Delta p} \right), 0 \right\}, \quad (15)$$

which is decreasing in  $R - \frac{B}{\Delta p}$ . The project will be funded as long as

$$\max \left\{ \frac{p_l B}{(\Delta p)^2} \frac{1-p_h}{(2-p_h)p_h}, \frac{r}{p_h} \right\} \leq \left( R - \frac{B}{\Delta p} \right). \quad (16)$$

2) If  $p_h < \frac{1}{2}$ , whether the lower bound on  $K_g$  is increasing or decreasing in  $\bar{K}$  depends on which constraints are binding.

i) If the IR constraints of the investors are the only constraints that bind, then  $K_g$  is always decreasing in  $\hat{K}$  and it is optimal to set

$$\hat{K} = \left( R - \frac{B}{\Delta p} \right), \quad (17)$$

which implies

$$K_g = \frac{r}{1-p_h} - p_h \left( R - \frac{B}{\Delta p} \right).$$

To satisfy ICG it has to be the case that

$$\left( R - \frac{B}{\Delta p} \right) \geq \frac{1}{p_h} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} \right],$$

and to satisfy IRG it has to be the case that

$$\frac{r}{p_h} \leq \left( R - \frac{B}{\Delta p} \right),$$

and, to have  $K_g \geq 0$ , we need  $\frac{r}{p_h(1-p_h)} \geq \left(R - \frac{B}{\Delta p}\right)$ . Then, we will be in this case if

$$\frac{r}{p_h(1-p_h)} \geq \left(R - \frac{B}{\Delta p}\right) \geq \max \left\{ \frac{1}{p_h} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} \right], \frac{r}{p_h} \right\}.$$

ii) If the IC constraints of the government bind, then  $K_g$  is decreasing in  $\hat{K}$  and

$$\hat{K} = 0, \tag{18}$$

which implies

$$K_g = \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right).$$

To satisfy IRG in this case, we need

$$(1-p_h) \frac{p_l B}{(\Delta p)^2} \leq \left(R - \frac{B}{\Delta p}\right),$$

and to have  $K_0 \geq 0$  it has to be the case that

$$\frac{p_l B}{(\Delta p)^2} \geq \left(R - \frac{B}{\Delta p}\right),$$

and, to satisfy the IR constraints of the investors, we need

$$\frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} \geq \left(R - \frac{B}{\Delta p}\right). \tag{19}$$

Then, we will be in this case if

$$(1-p_h) \frac{p_l B}{(\Delta p)^2} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h}.$$

iii) Finally, if both constraints bind at the same time, then

$$(2p_h - 1) \hat{K} + K_g = \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right),$$

and

$$\frac{r}{1-p_h} = p_h \hat{K} + K_g.$$

Then,

$$\hat{K} = \frac{1}{1-p_h} \left[ -\frac{p_l B}{(\Delta p)^2} + \frac{r}{1-p_h} + \left(R - \frac{B}{\Delta p}\right) \right], \tag{20}$$

and

$$K_g = \frac{p_h}{1-p_h} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{2p_h - 1}{p_h} \frac{r}{1-p_h} - \left(R - \frac{B}{\Delta p}\right) \right].$$

To be in this case, we need  $\hat{K} \in \left[0, R - \frac{B}{\Delta p}\right]$ , which is satisfied if

$$\frac{1}{p_h} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} \right] \geq \left(R - \frac{B}{\Delta p}\right) \geq \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h}.$$

which implies  $K_g > 0$  as long as  $\frac{1}{2} \frac{p_l B}{(\Delta p)^2} < \frac{r}{1-p_h}$ . Finally, for IRG to be satisfied requires  $\frac{r}{p_h} \leq \left(R - \frac{B}{\Delta p}\right)$ . Then, to be in this case we need

$$\max \left\{ \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h}, \frac{r}{p_h} \right\} \leq \left(R - \frac{B}{\Delta p}\right) \leq \frac{1}{p_h} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{r}{1-p_h} \right].$$

Then,

$$K_g = \max \left\{ \frac{r}{1-p_h} - p_h \left(R - \frac{B}{\Delta p}\right), \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right), \frac{p_h}{1-p_h} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{2p_h - 1}{p_h} \frac{r}{1-p_h} - \left(R - \frac{B}{\Delta p}\right) \right], 0 \right\}. \quad (21)$$

If

$$\max \left\{ \frac{r}{p_h}, (1-p_h) \frac{p_l B}{(\Delta p)^2} \right\} \geq \left(R - \frac{B}{\Delta p}\right) \quad (22)$$

then the project is not funded.

### Proof of Proposition 5 (RO vs. GO)

a. Using that  $\underline{\Gamma} = \frac{r}{p_h}$  and Eq. (12) it follows that the project is not undertaken even in the absence of moral hazard when  $\left(R - \frac{B}{\Delta p}\right) < \underline{\Gamma}$ .

b. i. Using the definition of  $\underline{\Gamma}$  and that  $\bar{\Gamma} = (1-p_h) \frac{p_l B}{(\Delta p)^2}$  in Eq. (16) we have that if

$$\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) < \frac{1}{(2-p_h)p_h} \bar{\Gamma},$$

the project is not funded in the presence of government moral hazard.

b.ii. From Eq. (14) it follows that the cross guarantees are maximal when  $\left(R - \frac{B}{\Delta p}\right) \geq \max \left\{ \underline{\Gamma}, \frac{1}{(2-p_h)p_h} \bar{\Gamma} \right\}$  if  $p_h > \frac{1}{2}$ .

c. i. Using the definition of  $\underline{\Gamma}$  and  $\bar{\Gamma}$  in Eq. (22) we have that if

$$\underline{\Gamma} \leq \left(R - \frac{B}{\Delta p}\right) < \bar{\Gamma},$$

the project is not funded in the presence of government moral hazard.

c.ii. From the characterization of the optimal contract in part 2 above and Eqs. (17), (18), and (20) it follows that optimal cross-guarantees are positive if and only if

$$\left(R - \frac{B}{\Delta p}\right) > \frac{\bar{\Gamma}}{1-p_h} - \frac{r}{(1-p_h)}$$

and that cross-guarantees are zero otherwise.

d. From Eqs. (15) and (21) and the characterization of the optimal contract above, it follows that when  $\left(R - \frac{B}{\Delta p}\right) > \frac{\underline{\Gamma}}{(1-p_h)}$ , government guarantees are not needed to fund the project when cross-guarantees are chosen optimally.  $\square$

## D Development Rights and General Obligation Financing

In this section we characterize the optimal contract for the model with development rights and cross-guarantees in Section 5.1. We also provide the proofs for the results in this section.

### Characterization of optimal contract

In this case, the government solves

$$\begin{aligned} & \max_{\alpha, K_g^i, \hat{R}_b^i} (p_h (R + \hat{R}) - 1) I \\ & K^i I^i \in \left[ 0, \left( R - \frac{B}{\Delta p} \right) I^j \right], \\ & \alpha \in [0, 1], K_g^i \geq 0, \hat{R}_b^i \in [0, \hat{R}] \end{aligned}$$

s.t.

$$\frac{r - p_h \hat{R}_b^i}{1 - p_h} \leq p_h K^a + K_g^a, \quad (\text{IRA-GO-DR})$$

$$\frac{r - p_h \hat{R}_b^i}{1 - p_h} \leq p_h K^b + K_g^b, \quad (\text{IRB-GO-DR})$$

$$(1 - p_h) (K_g^a \alpha + K_g^b (1 - \alpha)) \leq p_h (\alpha \bar{R}_g^a + (1 - \alpha) \bar{R}_g^b) - p_h (1 - p_h) (K^a \alpha + K^b (1 - \alpha)), \quad (\text{IRG-GO-DR})$$

$$p_h K^a - (1 - p_h) K^b \frac{1 - \alpha}{\alpha} + K_g^a \geq \frac{p_l R - p_h \bar{R}_g^a}{\Delta p}, \quad (\text{ICGA-GO-DR})$$

$$p_h K^b - (1 - p_h) K^a \frac{\alpha}{1 - \alpha} + K_g^b \geq \frac{p_l R - p_h \bar{R}_g^b}{\Delta p}, \quad \text{and} \quad (\text{ICGB-GO-DR})$$

$$\alpha I K_g^a + (1 - \alpha) I K_g^b \leq \bar{K}, \quad (\text{Fiscal-Constraint-GO-DR})$$

where  $\bar{R}_g^i = \left( R - \frac{B}{\Delta p} \right) + \min \left\{ \hat{R} - \hat{R}_b^i, \frac{B}{\Delta p} \right\}$ .

The budget constraint Fiscal-Constraint-GO-DR holds with equality in equilibrium. Therefore, one can rewrite the objective function as follows

$$(p_h (R + \hat{R}) - 1) \frac{\bar{K}}{\alpha K_g^a + (1 - \alpha) K_g^b}.$$

Analogous to the analysis in the case considered in the previous section, if both projects are undertaken it has to be the case that  $K_g^a = K_g^b = K_g$ . Then, the problem above becomes

$$\begin{aligned} & \max_{\alpha, K_g, \hat{R}_b^i} (p_h (R + \hat{R}) - 1) \frac{\bar{K}}{K_g} \\ & K^i I^i \in \left[ 0, \left( R - \frac{B}{\Delta p} \right) I^j \right], \\ & \alpha \in [0, 1], K_g \geq 0, \hat{R}_b^i \in [0, \hat{R}] \end{aligned}$$

s.t.

$$\begin{aligned}
\frac{r - p_h \hat{R}_b^a}{1 - p_h} &\leq p_h K^a + K_g, \\
\frac{r - p_h \hat{R}_b^b}{1 - p_h} &\leq p_h K^b + K_g, \\
p_h (1 - p_h) (K^a \alpha + K^b (1 - \alpha)) + (1 - p_h) K_g &\leq p_h (\alpha \bar{R}_g^a + (1 - \alpha) \bar{R}_g^b), \\
p_h K^a - (1 - p_h) K^b \frac{1 - \alpha}{\alpha} + K_g &\geq \frac{p_l R - p_h \bar{R}_g^a}{\Delta p}, \quad \text{and} \\
p_h K^b - (1 - p_h) K^a \frac{\alpha}{1 - \alpha} + K_g &\geq \frac{p_l R - p_h \bar{R}_g^b}{\Delta p}.
\end{aligned}$$

To satisfy the three IR constraints it must be the case that

$$\frac{r - p_h \hat{R}}{p_h} \leq \left( R - \frac{B}{\Delta p} \right).$$

As in the previous section, the individual rationality constraints of the investor and the incentive compatibility constraints of the government impose lower bounds on the government guarantee  $K_g$  as follows:

$$\begin{aligned}
K_g \geq \max \left\{ \frac{r - p_h \hat{R}_b^a}{1 - p_h} - p_h K^a, \frac{r - p_h \hat{R}_b^b}{1 - p_h} - p_h K^b, \frac{p_l R - p_h \bar{R}_g^a}{\Delta p} - \left( p_h K^a - (1 - p_h) K^b \frac{1 - \alpha}{\alpha} \right), \right. \\
\left. \frac{p_l R - p_h \bar{R}_g^b}{\Delta p} - \left( p_h K^b - (1 - p_h) K^a \frac{\alpha}{1 - \alpha} \right), 0 \right\}.
\end{aligned}$$

This constraint is minimized at  $\alpha = \frac{1}{2}$ , which implies

$$\begin{aligned}
K_g \geq \max \left\{ \frac{r - p_h \hat{R}_b^a}{1 - p_h} - p_h K^a, \frac{r - p_h \hat{R}_b^b}{1 - p_h} - p_h K^b, \frac{p_l R - p_h \bar{R}_g^a}{\Delta p} - \left( p_h K^a - (1 - p_h) K^b \right), \right. \\
\left. \frac{p_l R - p_h \bar{R}_g^b}{\Delta p} - \left( p_h K^b - (1 - p_h) K^a \right), 0 \right\}.
\end{aligned}$$

Moreover, the constraint is also minimized when  $K^a = K^b$ . Hence,

$$K_g \geq \max \left\{ \frac{r - p_h \min \{ \hat{R}_b^a, \hat{R}_b^b \}}{1 - p_h} - p_h \hat{K}, \frac{p_l R - p_h \min \{ \bar{R}_g^a, \bar{R}_g^b \}}{\Delta p} - (2p_h - 1) \hat{K}, 0 \right\},$$

where  $K^a = K^b = \hat{K}$  and

$$0 \leq \hat{K} \leq \left( R - \frac{B}{\Delta p} \right).$$

There are two cases, depending on whether  $p_h \gtrless \frac{1}{2}$ .

1) If  $p_h \geq \frac{1}{2}$ , the lower bound on  $K_g$  is always decreasing in  $\hat{K}$ . In this case, it is optimal to set

$$\hat{K} = \left( R - \frac{B}{\Delta p} \right), \quad (23)$$



and

$$K_g = \max \left\{ \frac{r - p_h \min \{ \hat{R}_b^a, \hat{R}_b^b \}}{1 - p_h} - p_h \left( R - \frac{B}{\Delta p} \right), \frac{p_l R - p_h \min \{ \bar{R}_g^a, \bar{R}_g^b \}}{\Delta p} - (2p_h - 1) \left( R - \frac{B}{\Delta p} \right), 0 \right\},$$

which is decreasing in  $R - \frac{B}{\Delta p}$ . As in the model with development rights only, the development rights will be fully assigned to the investors if and only if individual rationality constraint binds and the government's incentive compatibility constraints are slack, they will be fully assigned to the private sector if and only if the incentive compatibility constraints of the government are binding and the investors' individual rationality constraints are slack, and they will be shared between the two private parties if all four constraints bind. If the four constraints bind, it has to be the case that

$$\frac{r - p_h \hat{R}_b^i}{1 - p_h} - p_h \left( R - \frac{B}{\Delta p} \right) = \frac{p_l B}{(\Delta p)^2} - \frac{p_h}{\Delta p} \min \left\{ \hat{R} - \hat{R}_b^i, \frac{B}{\Delta p} \right\} - 2p_h \left( R - \frac{B}{\Delta p} \right)$$

If  $\hat{R} - \hat{R}_b^i \leq \frac{B}{\Delta p}$ , we have

$$\hat{R}_b^i = -\frac{1 - p_h}{1 - p_l} \frac{\Delta p}{p_h} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} - \frac{p_h}{\Delta p} \hat{R} - p_h \left( R - \frac{B}{\Delta p} \right) \right],$$

which implies that

$$K_g = \frac{r}{1 - p_h} + \frac{\Delta p}{1 - p_l} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} - \frac{p_h}{\Delta p} \hat{R} - p_h \left( R - \frac{B}{\Delta p} \right) \right] - p_h \left( R - \frac{B}{\Delta p} \right).$$

Note that if  $\hat{R} - \hat{R}_b^i > \frac{B}{\Delta p}$ , then  $\bar{R}_g = R$  and the government's moral hazard constraint is eliminated. In this case, the incentive compatibility constraint of the government is always satisfied when  $p_h > \frac{1}{2}$ , so we cannot have both constraints bind. Then

$$K_g = \max \left\{ 0, \frac{r - p_h \hat{R}}{1 - p_h} - p_h \left( R - \frac{B}{\Delta p} \right), \frac{p_l B}{(\Delta p)^2} - 2p_h \left( R - \frac{B}{\Delta p} \right) - \frac{p_h \hat{R}}{\Delta p}, \frac{r}{1 - p_h} + \frac{\Delta p}{1 - p_l} \left[ \frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - p_h} - \frac{p_h}{\Delta p} \hat{R} \right] - \frac{1 - p_l + \Delta p}{1 - p_l} p_h \left( R - \frac{B}{\Delta p} \right) \right\}.$$

Note that the expression for  $K_g$  above is decreasing in the cash flows from the development right  $\hat{R}$ . Then, development rights increase the size of the project when there are cross-guarantees.

Since the individual rationality constraint of the government has to be satisfied, the project will be funded with maximal cross-guarantees as long as

$$\max \left\{ \left( (1 - p_h) \frac{p_l B}{(\Delta p)^2} - (1 - p_l) \frac{p_h \hat{R}}{\Delta p} \right) \frac{1}{(2 - p_h) p_h}, \frac{r}{p_h} - \hat{R} \right\} \leq \left( R - \frac{B}{\Delta p} \right), \quad (24)$$

which is decreasing in  $\hat{R}$ . Hence, development rights make the project more likely to be funded with cross-guarantees.

If cross-guarantees are zero, then the project is funded whenever

$$\max \left\{ \left( (1 - p_h) \frac{p_l B}{(\Delta p)^2} - (1 - p_l) \frac{p_h \hat{R}}{\Delta p} \right), \frac{r}{p_h} - \hat{R} \right\} \leq \left( R - \frac{B}{\Delta p} \right). \quad (25)$$

Then, since  $(2 - p_h) p_h < 1$  we have that the project will be funded whenever the condition above is satisfied. Note that this region is increasing in  $\hat{R}$ .

2) If  $p_h < \frac{1}{2}$ , then the cross-guarantees will be positive and equal to  $\left(R - \frac{B}{\Delta p}\right)$  only if the individual rationality constraints of the investors are binding and the government's incentive compatibility constraints are slack, and they will be 0 if the incentive compatibility constraints of the government are binding and the investors' individual rationality constraints are slack. If the four constraints bind at the same time

$$\hat{K} = -\frac{1}{1-p_h} \left( \frac{p_l R - p_h \min\{\bar{R}_g^a, \bar{R}_g^b\}}{\Delta p} - \frac{r - p_h \min\{\hat{R}_b^a, \hat{R}_b^b\}}{1-p_h} \right),$$

which implies

$$K_g = \frac{1-2p_h}{1-p_h} \frac{r - p_h \min\{\hat{R}_b^a, \hat{R}_b^b\}}{1-p_h} + p_h \frac{1}{1-p_h} \left( \frac{p_l R - p_h \min\{\bar{R}_g^a, \bar{R}_g^b\}}{\Delta p} \right),$$

and  $\hat{R}_b^i$  will be chosen to minimize this expression. Then,

$$K_g \geq \max \left\{ \frac{r - p_h \hat{R}}{1-p_h} - p_h \left( R - \frac{B}{\Delta p} \right), \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right) - \frac{p_h \hat{R}}{\Delta p}, \right. \\ \left. \min_{\hat{R}_b \in [0, \hat{R}]} \left[ \frac{1-2p_h}{1-p_h} \frac{r - p_h \hat{R}_b}{1-p_h} - p_h \frac{1}{1-p_h} \left( \frac{p_l R - p_h \min\{\bar{R}_g^a, \bar{R}_g^b\}}{\Delta p} \right) \right], 0 \right\}.$$

The optimal cross-guarantees will be zero if and only if

$$(1-p_h) \left( \frac{p_l B}{(\Delta p)^2} - \frac{p_h \hat{R}}{\Delta p} \right) \leq \left( R - \frac{B}{\Delta p} \right) \leq \frac{p_l B}{(\Delta p)^2} - \frac{p_h \hat{R}}{\Delta p} - \frac{r}{1-p_h}. \quad (26)$$

Note that this set is decreasing with  $\hat{R}$ .

Finally, the project will be financed only if

$$\max \left\{ (1-p_h) \frac{p_l B}{(\Delta p)^2} - (1-p_l) \frac{p_h \hat{R}}{\Delta p}, \frac{r}{p_h} - \hat{R} \right\} \leq \left( R - \frac{B}{\Delta p} \right). \quad (27)$$

The lower bound for the project to be undertaken is decreasing in  $\hat{R}$ .

## Proof of Proposition 6

The proof of the proposition follows directly from noting that the lower bounds for  $K_g$  are decreasing in  $\hat{R}$  and that sets defined in Equations (24) and (25), and the set defined by the complements of Equations (26) and (27) are increasing in  $\hat{R}$ .

□

## ONLINE APPENDIX

### A Early-stage Government Moral Hazard

In this section we formalize the analysis of the model presented in Section 6. We first characterize the constraints faced by the government and private sector when the government faces moral hazard in an early stage and then characterize the optimal financing contract and provide comparative statics for the government guarantees.

#### Constraints with first-stage government moral hazard

The benchmark model in Section 3 is only played if the first stage succeeds. Therefore, the incentive compatibility constraints of the private sector and the government in the extortion stage remain unchanged. However, the input decision by the government in the first stage imposes an additional incentive compatibility constraint given by

$$e_h p_h \bar{R}_g - (1 - e_h) K_g^e - e_h (1 - p_h) K_g^p \geq b + e_l p_h \bar{R}_g - (1 - e_l) K_g^e - e_l (1 - p_h) K_g^p \quad (\text{ICG-GI-1})$$

or, using that  $\bar{R}_g = \left(R - \frac{B}{\Delta p}\right)$ ,

$$K_g^e \geq p_h \left[ \frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right) \right] + (1 - p_h) K_g^p.$$

If the government provides a high input, it gets a return  $\bar{R}_g$  if neither stage of the project fails, which occurs with probability  $e_h p_h$ , and pays guarantees if the project fails in either stage. The project fails in the first stage with probability  $(1 - e_h)$  and then the government pays  $K_g^e$  in guarantees per unit of investment; it fails with probability  $e_h (1 - p_h)$  in the second stage and, in this case, the government pays  $K_g^p$  in guarantees per unit of investment. If the government decides not to provide the high input, it gets a private benefit  $b$  and the probability of success in the first stage is  $e_l$ .

Note that the first and second-period guarantees have opposite effects on the government's incentives to provide input. On one hand, a higher first-stage guarantee increases the penalty for the government if the project fails in the first stage and thus increases the government's incentives to provide high input. On the other hand, a higher second-stage guarantee increases the penalty for the government if the project fails in the second stage, which decreases the government's expected payoff of the project upon succeeding in the first stage. Therefore, a higher second-stage guarantee exacerbates the government's moral hazard in the first stage.

The individual rationality constraints of the investors and the government also change to account for the two guarantees. In particular, the individual rationality constraint of the investors becomes

$$rI \leq (1 - e_h) K_g^e I + e_h (1 - p_h) K_g^p I + e_h p_h R_b I, \quad (\text{IRP-GI})$$

or using that the government will extort all the cash flow from the project from the investors, that is, that  $R_b = 0$ ,

$$r \leq (1 - e_h) K_g^e + e_h (1 - p_h) K_g^p.$$

Finally, the individual rationality constraint of the government is

$$0 \leq e_h p_h \bar{R}_g I - (1 - e_h) K_g^e I - e_h (1 - p_h) K_g^p I, \quad (\text{IRG-GI})$$

or using that  $\bar{R}_g = \left(R - \frac{B}{\Delta p}\right)$ ,

$$(1 - e_h) K_g^e + e_h (1 - p_h) K_g^p \leq e_h p_h \left(R - \frac{B}{\Delta p}\right).$$

## Characterization of optimal contract

When the government also faces moral hazard in the first stage, the contract solves the following problem

$$\max_{K_g^e, K_g^p \geq 0} (e_h p_h R - 1) \frac{\bar{K}}{\max\{K_g^e, K_g^p\}}$$

subject to

$$(1 - e_h) K_g^e + e_h (1 - p_h) K_g^p \geq r, \quad (\text{IRP-GI})$$

$$(1 - e_h) K_g^e + e_h (1 - p_h) K_g^p \leq e_h p_h \left( R - \frac{B}{\Delta p} \right), \quad (\text{IRG-GI})$$

$$K_g^e \geq p_h \left( \frac{b}{p_h \Delta e} - \left( R - \frac{B}{\Delta p} \right) \right) + (1 - p_h) K_g^p, \text{ and} \quad (\text{ICG-GI1})$$

$$K_g^p \geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right). \quad (\text{ICG-GI2})$$

The iso-curves of the objective function have a minimum at  $K_g^e = K_g^p$  and grow towards the origin. Then, the solution to the problem above will be a corner solution within the set of  $\{K_g^e, K_g^p\}$  that satisfies the four constraints above. There are four relevant cases to be considered, depending on which constraints bind.

1) Suppose that the individual rationality constraint for the private sector IRP-GI binds and all other constraints are slack. In this case,  $K_g^e = K_g^p = \bar{K}_g$ , where  $\bar{K}_g$  is given by

$$\bar{K}_g = \frac{r}{1 - e_h p_h}.$$

For the incentive compatibility constraints of the government to be satisfied we need the following two conditions to be satisfied

$$\frac{r}{1 - e_h p_h} \geq \frac{b}{p_h \Delta e} - \left( R - \frac{B}{\Delta p} \right)$$

and

$$\frac{r}{1 - e_h p_h} \geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right).$$

Finally, to satisfy the individual rationality constraint of the government it must be the case that

$$\frac{r}{e_h p_h} \leq \left( R - \frac{B}{\Delta p} \right) \quad (\text{IRG-GI})$$

Then,  $K_g^e = K_g^p = \bar{K}_g$  if

$$R - \frac{B}{\Delta p} \geq \max \left\{ \frac{b}{p_h \Delta e} - \frac{r}{1 - e_h p_h}, \frac{p_l B}{(\Delta p)^2} - \frac{r}{1 - e_h p_h}, \frac{r}{e_h p_h} \right\}.$$

2) Consider the case in which the the individual rationality constraint of the private sector binds and only one of the incentive compatibility constraints of the government binds and all the other constraints are slack.

i) If the moral hazard of the government in the first stage is more severe than the one in the second one, that is, if

$$\frac{p_l B}{(\Delta p)^2} < \frac{b}{p_h \Delta e}$$

and the incentive compatibility constraint for the government in the first stage ICG–GI1 binds, it has to be the case that  $K_g^e > K_g^p$ .

In this case, the contract guarantees are given by the solution to

$$(1 - e_h)K_g^e + e_h(1 - p_h)K_g^p = r$$

$$K_g^e + p_h \left( R - \frac{B}{\Delta p} \right) = \frac{b}{\Delta e} + (1 - p_h)K_g^p$$

Then,  $K_g^p$  is given by

$$K_g^p = \frac{r}{(1 - p_h)} - \frac{(1 - e_h)p_h}{(1 - p_h)} \left[ \frac{b}{p_h \Delta e} - \left( R - \frac{B}{\Delta p} \right) \right]$$

and  $K_g^e$  is

$$K_g^e = r + e_h \frac{b}{\Delta e} - e_h p_h \left( R - \frac{B}{\Delta p} \right)$$

$$K_g^e = r + e_h p_h \left[ \frac{b}{p_h \Delta e} - \left( R - \frac{B}{\Delta p} \right) \right].$$

For both guarantees to be positive it must be the case that

$$\frac{r}{e_h p_h} + \frac{b}{p_h \Delta e} \geq \left( R - \frac{B}{\Delta p} \right) \quad \text{and} \quad \left( R - \frac{B}{\Delta p} \right) \geq \frac{b}{p_h \Delta e} - \frac{r}{(1 - e_h) p_h}.$$

For the incentive constraint of the government in the second stage ICG–GI2 to be satisfied it must be the case that

$$K_g^p \geq \frac{p_l B}{(\Delta p)^2} - \left( R - \frac{B}{\Delta p} \right)$$

which is the same as

$$R - \frac{B}{\Delta p} \geq \frac{1}{1 - e_h p_h} \left[ (1 - p_h) \frac{p_l B}{(\Delta p)^2} - r + (1 - e_h) \frac{b}{\Delta e} \right].$$

To have  $K_g^p < K_g^e$  it has to be the case that

$$R - \frac{B}{\Delta p} < \frac{b}{p_h \Delta e} - \frac{r}{1 - e_h p_h}.$$

Hence, we will be in this case if

$$\max \left\{ \frac{r}{e_h p_h}, \frac{b}{p_h \Delta e} - \frac{r}{(1 - e_h) p_h}, \frac{1}{1 - e_h p_h} \left[ (1 - p_h) \frac{p_l B}{(\Delta p)^2} - r + (1 - e_h) \frac{b}{\Delta e} \right] \right\} \leq R - \frac{B}{\Delta p} < \frac{b}{p_h \Delta e} - \frac{r}{1 - e_h p_h}.$$

ii) If the moral hazard problem of the government in the second stage is more severe than the one in the first stage, that is, if

$$\frac{b}{p_h \Delta e} < \frac{p_l B}{(\Delta p)^2}$$

and the incentive compatibility constraint for the government in the second stage ICG–GI2 binds, it has to be the case that  $K_g^e < K_g^p$ .

In this case, the contract guarantees are given by the solution to

$$(1 - e_h)K_g^e + e_h(1 - p_h)K_g^p = r$$

$$K_g^p = \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right),$$

which implies

$$K_g^e = \frac{1}{(1 - e_h)} \left[ r - e_h(1 - p_h) \left[ \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) \right] \right].$$

To have both guarantees be greater than zero, we need

$$\frac{p_l B}{(\Delta p)^2} \geq \left(R - \frac{B}{\Delta p}\right) \geq \frac{p_l B}{(\Delta p)^2} - \frac{r}{e_h(1 - p_h)}.$$

For the incentive compatibility constraint of the government in the first stage to be satisfied, it has to be the case that

$$\left(R - \frac{B}{\Delta p}\right) \geq \frac{1}{1 - e_h p_h} \left[ (1 - p_h) \frac{p_l B}{(\Delta p)^2} - r + (1 - e_h) \frac{b}{\Delta e} \right].$$

To have  $K_g^e < K_g^p$  it has to be the case that

$$\left(R - \frac{B}{\Delta p}\right) < \frac{p_l B}{(\Delta p)^2} - \frac{r}{(1 - e_h p_h)}.$$

Then, we will be in this case if

$$\max \left\{ \frac{r}{e_h p_h}, \frac{p_l B}{(\Delta p)^2} - \frac{r}{e_h(1 - p_h)}, \frac{1}{1 - e_h p_h} \left[ (1 - p_h) \frac{p_l B}{(\Delta p)^2} - r + (1 - e_h) \frac{b}{\Delta e} \right] \right\} < R - \frac{B}{\Delta p} < \frac{p_l B}{(\Delta p)^2} - \frac{r}{(1 - e_h p_h)}.$$

3) Finally, if both incentive compatibility constraints of the government are binding and all other constraints are slack, the optimal guarantees are given by

$$K_g^p = \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)$$

$$K_g^e = p_h \left( \frac{b}{p_h \Delta e} \right) + (1 - p_h) \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right).$$

To have both guarantees be positive we need

$$\left(R - \frac{B}{\Delta p}\right) \leq \min \left\{ \frac{p_l B}{(\Delta p)^2}, p_h \left( \frac{b}{p_h \Delta e} \right) + (1 - p_h) \frac{p_l B}{(\Delta p)^2} \right\}$$

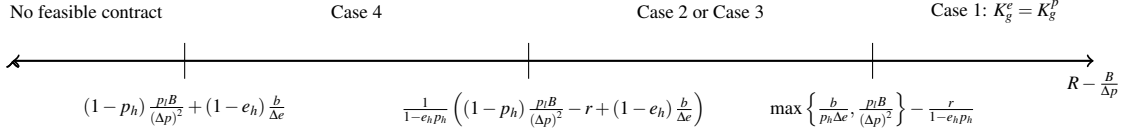


Figure A.1

For the individual rationality constraint of the private sector to be satisfied it has to be the case that

$$\left(R - \frac{B}{\Delta p}\right) \leq \frac{1}{(1 - e_h p_h)} \left[ (1 - p_h) \frac{p_l B}{(\Delta p)^2} - r + p_h (1 - e_h) \frac{b}{p_h \Delta e} \right].$$

For the individual rationality constraint of the government to be satisfied it has to be the case that

$$\left(R - \frac{B}{\Delta p}\right) \geq (1 - p_h) \frac{p_l B}{(\Delta p)^2} + p_h (1 - e_h) \frac{b}{p_h \Delta e}.$$

Then, we will be in this case if

$$\max \left\{ \frac{r}{e_h p_h}, (1 - p_h) \frac{p_l B}{(\Delta p)^2} + p_h (1 - e_h) \frac{b}{p_h \Delta e} \right\} \leq \left(R - \frac{B}{\Delta p}\right)$$

and

$$\left(R - \frac{B}{\Delta p}\right) \leq \min \left\{ \frac{p_l B}{(\Delta p)^2}, p_h \left( \frac{b}{p_h \Delta e} \right) + (1 - p_h) \frac{p_l B}{(\Delta p)^2}, \frac{1}{(1 - e_h p_h)} \left[ (1 - p_h) \frac{p_l B}{(\Delta p)^2} - r + (1 - e_h) \frac{b}{\Delta e} \right] \right\}.$$

Note that  $K_g^{e*} > K_g^{p*}$  if and only if  $\frac{p_l B}{(\Delta p)^2} < \frac{b}{p_h \Delta e}$ . ■

The figure below summarizes the four possible cases depending on the value of  $X = R - \frac{B}{\Delta p}$ .

## Optimal Guarantees

The optimal guarantees are given by the table below.

case	1	2	3	4
$K_g^e$	$\frac{r}{1 - e_h p_h}$	$r + e_h p_h \left( \frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right) \right)$	$\frac{1}{(1 - e_h)} \left( r - e_h (1 - p_h) \left( \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) \right) \right)$	$p_h \left( \frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right) \right) + (1 - p_h) \left( \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right) \right)$
$K_g^p$	$\frac{r}{1 - e_h p_h}$	$\frac{r}{(1 - p_h)} - \frac{(1 - e_h) p_h}{(1 - p_h)} \left( \frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right) \right)$	$\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)$	$\frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)$

As it can be seen from this table, the optimal guarantees depend on the type of moral hazard that binds and its severity. In case 1, the incentive compatibility constraints for the government are not binding and, therefore, the optimal guarantees are independent of the intensity of the moral hazard. In this case, the return of the project net of the required return of the private sector to exert effort is large enough that the government always prefers to exert effort and not to extort the private sector's return in the second stage.

In case 2, the moral hazard of the government in the first stage is more severe than its moral hazard in the second stage, and it is severe enough so that the incentive compatibility constraint of the government ICG-GII binds. In this case, the optimal guarantees are determined by the intensity of the governments

moral hazard in the first stage,  $Y \equiv \frac{b}{p_h \Delta e} - \left(R - \frac{B}{\Delta p}\right)$ . The higher  $Y$ , the more costly it is to incentivize the government to exert effort in the first stage. Therefore, the optimal guarantee in the first stage is increasing in  $Y$ . On the other hand, the government's guarantee in the second stage decreases in  $Y$  to satisfy the investors' individual rationality constraint with equality.

In case 3, the government's moral hazard in the second stage is more severe than its moral hazard in the first stage, and the incentive compatibility of the government ICG-GI2 binds. In this case, the optimal guarantees depend on the intensity of the government's moral hazard in the second stage,  $Z \equiv \frac{p_l B}{(\Delta p)^2} - \left(R - \frac{B}{\Delta p}\right)$ . The higher  $Z$ , the harder it is to incentivize the government not to extort the private sector's return. In this case, the guarantee in the second stage is increasing in  $Z$ , while the guarantee in the first stage is decreasing in  $Z$ .

Finally, in case 4 both incentive compatibility constraints for the government bind. In this case, the optimal guarantee in the first stage depends positively on both intensities  $Y$  and  $Z$ , while the optimal guarantee in the second stage only depends on  $Z$  positively.

The intensity of the moral hazard of the private sector determines the return of the government in the event that the project succeeds. Hence, since the intensity of the government's moral hazard problems is measured relative to this return, the optimal guarantees also depend on the intensity of the private sector's moral hazard, measured by  $-X$ . Note that the higher  $X$ , the less severe the moral hazard of the private sector. The optimal guarantees are such that the maximum guarantee is always (weakly) decreasing in  $X$ .

These comparative statics are given by the following table and summarized in the proposition below.

case	1: $K_g^e = K_g^p$	2: $K_g^e > K_g^p$	3: $K_g^e < K_g^p$	4: $K_g^e \leq K_g^p$ $K_g^e \geq K_g^p$
$\frac{dK_g^e}{dX}$	0	$-e_h p_h$	$\frac{e_h}{1-e_h} (1-p_h)$	-1
$\frac{\partial K_g^e}{\partial Y}$	0	$e_h p_h$	0	$p_h$
$\frac{\partial K_g^e}{\partial Z}$	0	0	$-\frac{e_h}{1-e_h} (1-p_h)$	$(1-p_h)$
$\frac{dK_g^p}{dX}$	0	$(1-e_h) \frac{p_h}{1-p_h}$	-1	-1
$\frac{\partial K_g^p}{\partial Y}$	0	$-(1-e_h) \frac{p_h}{1-p_h}$	0	0
$\frac{\partial K_g^p}{\partial Z}$	0	0	1	1